Bargaining and waning commitments

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Abstract

This paper presents a non-cooperative two-agent, two-period complete information bargaining model that introduces gradual concession (as observed, for example, in labor negotiations). The capacity of concession may affect the agents’ payoffs in a non-monotonic way since its change may cause the timing of agreement to change.

Keywords: Bilateral bargaining; Subgame perfect Nash equilibrium

JEL classification: C78; C72

1. Introduction

In many bargaining situations the negotiators take actions that commit them to some bargaining positions. Schelling (1980) pointed out that commitment is an important factor in determining bargaining outcomes. Since then, many papers have studied the effect of such tactics in bargaining games (see related work below).

As shown by the experiments in behavior economics, people are often more concerned with the changes than with the absolute values. Hence, if the findings in the experiments are correct, the concession is likely to be proportional to the standing offer. For example, labor economists have long recognized the occurrence of gradual concession (decreasing commitment) in labor negotiations (Ashenfelter and Johnson, 1969, argued that strikes occur because the union concedes gradually).

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In this paper, we construct a simple two-period, sequential-move game that captures this notion of (possibly) decreasing commitments over time. Each bargainer is characterized by two parameters: the discount factor and the oblivion factor (the rate at which the commitments may decrease). We show that, for some values of the parameters, the only subgame perfect Nash equilibrium outcome exhibits agreements which are delayed until the last period. We analyze how the equilibrium changes when the parameters change. We show that the parameters may affect the agents’ payoffs in a non-monotonic way since the change in the parameters may cause the timing of agreement to change.

1.1. Related work

Some works have analyzed models with partial commitments, where commitments in the first period can be revoked at a cost in the second period. Our paper is related to this literature since, in some sense, we assume that the cost of revoking commitment is zero up to a certain point, and is infinity beyond that point. For example, Crawford (1982) analyzed a model where the bargainers can make revocable commitments and showed that, when commitments are uncertain, the bargainers may reach an impasse. In contrast to that paper, however, we do not need uncertainty to show that inefficient outcomes can arise in equilibrium. Other works, e.g. Muthoo (1992, 1996), have analyzed models with partial commitments in which bargaining outcomes are efficient. These models are different from ours in two important elements: the bargainers chose their commitments simultaneously, and the cost to a bargainer of revoking a commitment is proportional to the amount by which he has deviated from his commitment.

2. The bargaining model

Two societies, $A$ and $B$, have to decide how to split one dollar. Bargaining will be conducted between two agents representing these societies: agent $A$ and agent $B$, respectively. An agent’s interests is assumed to coincide with those of the society that he represents. The negotiation proceeds in two periods, $t=1, 2$. We consider the following four-stage game:

1. In the first period, agent $A$ announces the minimum share that he would accept in that period: $x_A^1 \in [0, 1]$. We call this announcement the commitment for the first period of agent $A$.
2. Agent $B$ observes $x_A^1$ and announces his commitment for the first period: $x_B^1 \in [0, 1]$. If $x_A^1 + x_B^1 \leq 1$, the commitments are called compatible and then the agents (societies) split the dollar at the middle of their commitments (giving $x_i^1 + (1 - x_i^1 - x_B^1)/2$ to each agent $i \in \{A, B\}$) and the game ends. Otherwise, nothing happens until the second period.
3. In the second period, agent $A$ observes $x_B^1$ and announces his commitment for the second period: $x_A^2$.

We will interpret the announcement $x_A^2$ as the possibility of agent $A$ to reaffirm publicly the commitment of the first period with varying intensity. At the very most, agent $A$ can stand firm on his initial commitment. If agent $A$ does not reaffirm his initial commitment at all, his society will “forget”

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1 Other works analyzing the influence of various commitment tactics on the efficiency of the bargaining outcomes are Dekel (1990), Fershtman and Seidmann (1993), Abreu and Gul (1997), and Cai (2000), among others.

2 Other works studying the influence of commitment tactics in which the bargaining outcomes are efficient are, e.g., Osborne and Rubinstein (1990), and Kambe (1999).
it gradually. In this case, the initial commitment will be discounted by an oblivion factor \( \lambda_A \in (0, 1) \). Then \( x_A^2 \) can take any value in the set \([\lambda_A x_A^1, x_A^1]\).  

4. Agent B observes \( x_A^2 \) and announces his commitment for the second period \( x_B^2 \in [\lambda_B x_B^1, x_B^1] \) (where \( \lambda_B \in (0, 1) \) is society B’s oblivion factor). If \( x_A^2 + x_B^2 \), each agent gets \( x_A^2 + (1 - x_A^2 - x_B^2)/2 \). Otherwise, each agent gets a payoff equal to zero.

Each agent \( i \) has a discount factor \( \delta_i \in (0, 1) \) between the first and the second period. An environment is a profile \( e = ((\lambda_A, \delta_A); (\lambda_B, \delta_B)) \in (0, 1)^2 \times (0, 1)^2 \). Each agent knows both \((\lambda_A, \delta_A)\) and \((\lambda_B, \delta_B)\). Let \( E = (0, 1)^2 \times (0, 1)^2 \) be the set of feasible environments. For all \( e \in E \), \( G(e) \) is the bargaining game in its extensive form.

Our equilibrium concept is a subgame perfect Nash equilibrium (SPE). We denote an agreement in period \( t \) as \( z_t \in [0, 1] \), so that agent \( A \)'s share is \( z_t \) and agent \( B \)'s share is \((1 - z_t)\). For all \( e \in E \) and all \( t \in \{1, 2\} \), let \( Z(e) \) be the set of agreements in period \( t \) that are a SPE outcome of game \( G(e) \).

3. The results

We begin at the fourth stage of the game. At that stage, agent B announces the largest feasible commitment among those which make an agreement possible (given \( x_A^2 \)). If this is not possible, he announces any feasible commitment. Then, B’s best reply correspondence in the fourth stage is:

\[
x_B^2(x_A^1, x_B^1, x_A^2) = \begin{cases} 
\min \{1 - x_A^2, x_B^1\} & \text{if } x_A^2 \leq 1 - \lambda_B x_B^1 \\
[\lambda_B x_B^1, x_B^1] & \text{otherwise}
\end{cases}
\]

(1)

At the third stage, agent A announces the largest feasible commitment among those which make it possible for B to announce a compatible commitment. If this is not possible, A announces any feasible commitment. Then, A’s best reply correspondence in the third stage is:

\[
x_A^2(x_A^1, x_B^1) = \begin{cases} 
\min \{x_A^1, 1 - \lambda_B x_B^1\} & \text{if } \lambda_A x_A^1 + \lambda_B x_B^1 \leq 1 \\
[\lambda_A x_A^1, x_A^1] & \text{otherwise}
\end{cases}
\]

(2)

In the second stage of the game, given \( x_A^1 \), agent B has to decide between bringing about an agreement in the first period or in the second period. In the first case he would announce \( x_B^1 = 1 - x_A^1 \), and they would reach an agreement \( z^2 = x_A^1 \). In the second case, he would announce the largest commitment among those which make it possible to reach an agreement in the second period, i.e., \( x_B^1 = \min \{1 - \lambda_B, 1 - \lambda_A x_A^1\} \). In this case, from (1) and (2), the agents would reach an agreement \( z^2 = \min \{x_A^1, \max \{1 - \lambda_B, 1 - \lambda_A x_A^1\}\} \) in the second period. Then, B’s best reply correspondence in the second stage is:

\[
x_B^1(x_A^1) = \begin{cases} 
\min \{1, 1 - \lambda_A x_A^1\} & \text{if } \delta_B \left( \max \{1 - x_A^1, \min \{1 - \lambda_B, 1 - \lambda_A x_A^1\}\} \right) > 1 - x_A^1 \\
1 - x_A^1 & \text{otherwise}
\end{cases}
\]

(3)

In the first stage we distinguish two cases according to the environment. In particular, we show that if \( \max \{1 - \delta_B, 1 - \lambda_B, \lambda_A x_A^1\} \) then the commitment for the first period announced by agent A will bring

\[3\]  To allow \( x_A^2 > x_A^1 \) would not change the main results.

\[4\]  We also assume that no time elapses between the first (resp. third) and the second (resp. fourth) stages.
about an agreement in the first period. If \( \max \left\{ \frac{1-\delta_B}{1-\delta_B A}, 1 - \delta_B \lambda_B \right\} \geq \delta_A \lambda_A \), however, agent A will prefer to announce a commitment for the first period that brings about a delayed (and therefore inefficient) agreement.

**Theorem 1.** Let \( e \in E \) be such that \( \max \left\{ \frac{1-\delta_B}{1-\delta_B A}, 1 - \delta_B \lambda_B \right\} \geq \delta_A \lambda_A \). Then \( Z^1(e) = \max \left\{ \frac{1-\delta_B}{1-\delta_B A}, 1 - \delta_B \lambda_B \right\} \) and \( Z^2(e) = \emptyset \).

**Proof.** In the first stage of the game, agent A has to decide between bringing about an agreement in the first or in the second period. The best agreement in the second period that can be reached by A is \( z^2 = \lambda_A \) (for that, he should announce \( x_A^1 = 1 \)). The best agreement in the first period that can be brought about by A is \( z^1 = \lambda_A \). The agents will reach an agreement in the first period if and only if \( \delta_B(\max\{1-x_A^1, \min\{\lambda_B, 1-\lambda_A x_A^1\}\}) \leq 1-x_A^1 \) (in that case, the agents will reach an agreement \( z^1 = x_A^1 \)). Then, in order to bring about the best agreement in the first period, A should announce the largest \( x_A^1 \) satisfying either (i) \( x_A^1 \leq \min\{\frac{1-\delta_B}{1-\delta_B A}, 1 - \delta_B \lambda_B \} \), or (ii) \( \frac{1-\delta_B}{1-\delta_B A} \leq 1 - \delta_B \lambda_B \). For all \( e \in E \), either (a) \( \frac{1-\delta_B}{1-\delta_B A} \leq 1 - \delta_B \lambda_B \), or (b) \( \frac{1-\delta_B}{1-\delta_B A} \geq 1 - \delta_B \lambda_B \). Therefore, in Case (a) A should announce \( x_A^1 = \frac{1-\delta_B}{1-\delta_B A} \), and in Case (b) A should announce \( x_A^1 = 1 - \delta_B \lambda_B \). Hence, the best agreement in the first period that can be brought about by A is \( z^1 = \max \left\{ \frac{1-\delta_B}{1-\delta_B A}, 1 - \delta_B \lambda_B \right\} \). Since \( \max \left\{ \frac{1-\delta_B}{1-\delta_B A}, 1 - \delta_B \lambda_B \right\} \geq \delta_A \lambda_A \), agent A will reach an agreement in the first period.

**Theorem 2.** Let \( e \in E \) be such that \( \max \left\{ \frac{1-\delta_B}{1-\delta_B A}, 1 - \delta_B \lambda_B \right\} \geq \delta_A \lambda_A \). Then \( Z^1(e) = \emptyset \) and \( Z^2(e) = \lambda_A \).

**Proof.** As we have shown in the proof of Theorem 1, the best agreements in the first and second period that can be brought about by A are \( z^1 = \max \left\{ \frac{1-\delta_B}{1-\delta_B A}, 1 - \delta_B \lambda_B \right\} \) and \( z^2 = \lambda_A \), respectively. Since \( \max \left\{ \frac{1-\delta_B}{1-\delta_B A}, 1 - \delta_B \lambda_B \right\} \delta_A \lambda_A \), agent A will announce \( x_A^1 = 1 \) to bring about an agreement in the second period.

The reason that the equilibrium outcome sometimes entails delays is that, when the discount factors are sufficiently close to one, the agents may want to wait for the opponents’ concessions. For example,
given $x_A^1$, if agent $B$ announces $x_B^1 = 1 - \frac{1 - \lambda_A^1}{\lambda_B}$ (when possible), then in the second period agent $A$ will make the necessary concession to reach an agreement and will reduce his demand to $x_A^2 = \lambda_A x_A^1 < x_A^1$.

From Theorems 1 and 2 we obtain the following conclusions on the effect of changes in the oblivion factors:

Changes in $k_A$: Given some fixed values for $d_A$, $d_B$, and $k_A$, increases in $k_A$ generate two different effects. On the one hand, agent $A$ can guarantee himself a better agreement in the second period. On the other hand, agent $B$ will be willing to accept worse agreements in the first period, since the second period becomes less attractive for him. Depending on which effect prevails, the SPE agreement will take place in the second or first period. Fig. 1 shows an example of the effect of changes in $k_A$ on the SPE agreement. If $k_A^* < \lambda_A$, the SPE agreement, $z^1 = 1 - \delta_B \lambda_B$, takes place in the first period. If $k_A$ increases so that $k_A^* < \lambda_A < k_A^{**}$, then the SPE agreement, $z^2 = \lambda_A$, will move to the second period. Finally, if $k_A^{**} > \lambda_A$, then the SPE agreement, $z^1 = 1 - \delta_B \lambda_B$, goes back to the first period. Fig. 1 also shows agents’ payoffs in terms of $\lambda_A$ (the solid curve represents $A$’s payoffs and the dotted curve represents $B$’s payoffs). The best thing for agent $A$ would be $k_A$ was as large as possible. Regarding agent $B$, the best thing would be $k_A$ was smaller than $k_A^*$. Note that $B$’s payoffs are affected in a non-monotonic way by $k_A$, since $\delta_B (1 - \lambda_A^*) < \delta_B \lambda_B$ and $\delta_B (1 - \lambda_A^{**}) < 1 - \frac{1 - \delta_B}{1 - \delta_B}$.7

Changes in $k_B$: Given some fixed values for $\delta_A$, $\delta_B$, and $\lambda_A$, increases in $k_B$ make SPE agreements in the second period more likely. Consider the example in Fig. 2. If $k_B^* < k_B^{**}$ then the SPE agreement, $z^1 = 1 - \delta_B \lambda_B$, takes place in the first period, and if $k_B^{**} < \lambda_B$ then the SPE agreement, $z^2 = \lambda_A$, takes place.

5 Regarding the discount factors, it can be shown that (when $\lambda_A + \lambda_B > 1$) as $\delta_A$ or $\delta_B$ increases, it is more likely that the SPE agreement will take place in the second period.

6 The example illustrated in Fig. 1 corresponds with the case in which $\frac{1 - \delta_B \lambda_B}{\delta_A} < \frac{1 - \delta_B}{1 - \delta_B \lambda_B}$, if $\frac{1 - \delta_B \lambda_B}{\delta_A} > \frac{1 - \delta_B}{1 - \delta_B \lambda_B}$, then the SPE agreement always takes place in the first period.

7 Since $1 - \delta_B \lambda_B - \delta_A \lambda_B^* > 1$, $\lambda_A^* + \lambda_B$, and therefore $\delta_B (1 - \lambda_A^*) < \delta_B \lambda_B$.  

Fig. 2. The effect of changes in $\lambda_B$ on the timing of the agreement and on agents’ payoffs.
in the second period. Fig. 2 also shows agents’ payoffs in terms of $\lambda_B$. The best thing for agent $A$ would be $\lambda_B$ was as small as possible. Regarding agent $B$, however, the best thing would be that $\lambda_B$ was the largest among those guaranteeing an agreement in the first period, i.e., $\lambda_B = \lambda_B^*$, in which case the SPE agreement would be $z_1 = \delta_B \lambda_B^*$ ($B$’s payoffs are affected in a non-monotonic way by $\lambda_B$, since $\delta_B(1 - \lambda_A) < \delta_B \lambda_B^*$).

4. Conclusion

We study a relatively simple bargaining model with complete information which introduces gradual concession. We show that the agents’ capacity for concession affects their payoffs in a non-monotonic way since its variation may cause inefficient outcomes to arise in equilibrium.

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References


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8 In the example depicted in Fig. 2 $\frac{1-\delta_B}{1-\delta_B \lambda_A} < \delta_A \lambda_A$. If $\delta_A \lambda_A < \frac{1-\delta_B}{1-\delta_B \lambda_A}$ then the SPE agreement always takes place in the first period.