Heat and mass transfer in annular liquid jets: II. g-jitter

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Abstract

The effects of g-jitter on heat and mass transfer in underpressurized, annular liquid jets are analyzed numerically as a function of the amplitude and frequency of the gravitational modulation by means of a mapping technique that transforms the time-dependent geometry of these jets into a unit square and a conservative finite difference method. It is shown that the pressure coefficient, gas concentration at the jet’s inner interface, heat fluxes at the jet’s inner and outer interfaces and interfacial temperature are periodic functions of time whose amplitudes increase as the amplitude of the g-jitter is increased, but decrease as the jitter frequency is increased. The pressure coefficient is almost in phase with the heat flux at the jet’s outer interface, and out of phase with the mass transfer rate at the jet’s inner interface. It is also shown that the temperature field adapts itself rapidly to the imposed gravity modulation, and thermal equilibrium is reached quickly. However, mass transfer phenomena are very slow and require a very long time to become periodic. © 2000 Elsevier Science Inc. All rights reserved.

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1. Introduction

In Part I [1] of this series of papers dealing with heat and mass transfer phenomena in annular liquid jets, a high Reynolds number model was developed to study numerically the unsteady dynamics of annular jets when, starting...
from an initial steady condition corresponding to injection of gases into the volume enclosed by the jet identical to the heat and mass absorption rates by the liquid, the injection was stopped. It was found that, due to the small binary diffusion coefficient of gases in liquids, the mass absorption rate was smaller than the heat transfer one. As a consequence, the time required by both the flowing liquid and the gases enclosed by the jet to reach thermal equilibrium was shorter than that corresponding to zero mass transfer rates.

In Part [1], it was also shown that the heat and mass transfer processes in annular liquid jets exhibit stiffness in both space and time due to the initial rapid cooling of the liquid and the thin mass and energy boundary layers that are formed at the jet’s inner interface. In this paper, the effects of gravity modulation, i.e., temporal variations of the gravitational acceleration, on heat and mass transfer phenomena in annular liquid jets in the absence of combustion within the volume enclosed by these jets is studied by means of the formulation presented in Part I.

Residual acceleration or gravitational modulation is of great importance in microgravity environments where the sources of residual acceleration range from the effects of the earth’s gravity gradient to $g$-jitter accelerations which include atmospheric drag on the spacecraft, vibrations of compressors, spacecraft attitude motions arising from machinery vibrations, thruster firings, crew motion, etc. [2]. High frequency accelerations are, in general, unimportant compared with the residual motions caused by low frequency accelerations [3].

Gravity modulation has large effects on materials processing in space or in gravity-reduced environments [2,4]. For example, Alexander et al. [4] found that the orientation of the residual gravity is a crucial factor in determining the suitability of the spacecraft environment as a means to suppress or eliminate unwanted effects caused by buoyant fluid motion in Bridgman’s crystal growth. These authors also found that $g$-jitter affects the compositional uniformity of the growing crystal.

Gravity modulation also affects the stability of heated fluid layers. In the linear stability studies of Gresho and Sani [5], it was found that a sinusoidal modulation of the gravitational field in a horizontal layer of fluid heated from above or below can significantly affect the stability boundaries of the layer. Kamotani et al. [3] showed that, under normal circumstances, i.e., no maneuvers, no intentional spinning of the spacecraft, etc., $g$-jitter generates predominantly oscillatory velocity and temperature fields with zero time-mean value, but gravity modulation can also generate secondary flows with nonzero mean of much smaller magnitude. Tsau et al. [6] studied the effects of $g$-jitter on a buoyant flow in a cylindrical container heated from above, and found that the imposed gravitational acceleration causes a periodic flow field and heat transfer enhancement. They also found that the modulations of the three components of the gravitational field contribute to the heat transfer increase but in different ways, and that, sometimes, the flow field cannot remain
quasisteady and hot, isolated packets of fluid may appear away from the heat source.

The secondary flows observed by Kamotami et al. [3] have also been found by Farooq and Homsy [7] who investigated the streaming due to $g$-jitter-induced natural convection in a square cavity where a lateral temperature gradient interacts with a constant gravity field modulated by small harmonic oscillations, and found that a regular perturbation method based on the amplitude of the gravitational modulation results in Reynolds-stress-type terms that cause streaming. The streaming flow may have a strong influence on the strength of the recirculation flow pattern and the overall heat transfer rate; they also found that the gravitational modulation may interact with the flow instabilities and yield parametric resonances.

The streaming flows generated by gravity modulation are similar to those found in acoustics [8,9], and unsteady flows at stagnation points produced by outer, inviscid oscillatory flows of zero-mean [10,11]. For annular liquid jets, Falgueras and Ramos [12] showed that sinusoidal fluctuations in the liquid’s flow rate at the nozzle exit produce oscillatory pressure coefficients and periodic variations in the volume enclosed by the jet whose amplitude and frequency increase as those of the imposed flow rate are increased. Ramos [13] studied the effects of $g$-jitter in annular liquid jets in the absence of mass transfer and found that the fluid dynamics of these jets exhibits periodic and

Fig. 1. Schematic representation of an annular liquid jet.
quasiperiodic motions and broad spectra as the frequency of the residual acceleration is increased. Ramos [14] analyzed numerically the effects of g-jitter in annular liquid jets with mass transfer and found that the pressure coefficient, volume of the gases enclosed by the jet, interfacial gas concentration and mass absorption rate at the jet’s inner interface are periodic functions whose amplitudes increase as the amplitude of the g-jitter is increased.

For experiments carried aboard a spacecraft, the residual acceleration is, in general, three-dimensional. In this paper, however, we neglect the earth’s gravity gradient since the convergence length of the annular liquid jets considered here is small compared with the characteristic dimensions of the spacecraft, assume that the g-jitter accelerations are axial and consider heat and mass transfer between the gases enclosed by the jet and the flowing liquid. These assumptions allow us to consider axisymmetric, annular jets. Furthermore, since the equations which govern the fluid dynamics of heat and mass transfer in annular liquid jets subject to axial g-jitter accelerations in inertial frames of reference are analogous to those in noninertial frames which accelerate in a direction parallel to inertial ones, the study presented here is also valid for noninertial frames of reference, e.g., the nozzle exit shown in Fig. 1 translates along the x-axis with a time-dependent acceleration. In both cases, the numerical study presented here may be considered as an analysis of the effects of fluctuating body forces on heat and mass transfer phenomena in annular liquid jets in the absence of combustion.

g-jitter accelerations may not be periodic functions of time. In this study, however, it is assumed that they are sinusoidal functions of time. Since the equations governing the fluid dynamics of and heat and mass transfer in annular liquid jets have been presented in Part I [1], only a summary of these equations appears in Section 2. This summary is aimed at illustrating both the nonlinear coupling between the linear momentum and heat and mass transfer equations in time-dependent annular liquid jets, and the effects of g-jitter on the fluid dynamics equations. In the presence or absence of g-jitter, the fluid dynamics and gas concentration equations have been solved numerically by means of a domain-adaptive finite difference method which maps the curvilinear geometry of the jet into a unit square [15]. Section 3 shows the effects of the amplitude and frequency of the g-jitter on the fluid dynamics of and heat and mass transfer in annular liquid jets. The effects of the other nondimensional parameters which characterize the fluid dynamics and heat and mass transport in annular jets have been studied in Part I [1] in the absence of g-jitter, and are not considered here.

2. Governing equations

The nondimensional equations that govern the fluid dynamics of and mass and energy transfer in annular liquid jets can be written as follows [1].
Gas concentration in the liquid

\[
\frac{\partial \rho_j}{\partial t} + u \frac{\partial \rho_j}{\partial x} + v \frac{\partial \rho_j}{\partial r} = \frac{D_L^*}{D_{L,j}^*} \frac{1}{\text{Pe}_M} \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \rho_j}{\partial r} \right),
\]

(1)

\[ \rho_j(r, 0, t) = \alpha(\beta_j - 1), \]

(2)

\[ \rho_j(R_1(t, x), x, t) = \alpha \left( \frac{S_{i,j}^* p_{i,j}^*}{c_{eqe}^*} - 1 \right), \]

(3)

\[ \rho_j(R_2(t, x), x, t) = \alpha \left( \frac{S_{e,j}^* p_{e,j}^*}{c_{eqe}^*} - 1 \right), \]

(4)

where

\[ \alpha = c_{eqe}^*/\Delta c^*, \quad \beta_j = \rho_{0,j}^*/\Delta c^*, \]

(5)

\[ \Delta c^* = p_e^*/2R_T^* T_e^*, \]

(6)

\( \rho_j \) is the density of the \( j \)th gaseous species in the liquid, \( c_{eqe}^* \) is a reference concentration, \( t \) is time, \( u \) and \( v \) are the liquid’s axial and radial velocity components, respectively, \( x \) and \( r \) are the axial and radial coordinates, respectively, \( D_L^* \) is a reference binary diffusion coefficient of the gases in the liquid, \( D_{L,j}^* \) is the (constant) binary diffusion coefficient of the gaseous \( j \)th species in the liquid, \( \text{Pe}_M = u_0^* R_0^*/D_L^* \) is the mass Peclet number, \( u_0^* \) and \( R_0^* \) denote the jet’s (constant) axial velocity component and mean radius at the nozzle exit, respectively, \( R_1 \) and \( R_2 \) are the radii of the jet’s inner and outer interfaces, respectively, the subscripts \( i \) and \( e \) denote the gases enclosed by and surrounding the jet, respectively, \( S^* \) is Henry’s solubility, \( p \) is the pressure, \( T \) is the temperature, \( R_T \) is the specific gas constant, the subscript \( 0 \) denotes the nozzle exit and the asterisks denote dimensional quantities.

Temperature of the liquid jet

\[
\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial r} = \frac{1}{\text{Pe}_T} \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right),
\]

(7)

\[ T(R_1, x, t) = \frac{T_{i}^*}{T_e^*}, \quad T(R_2, x, t) = 1, \]

(8)

where \( \text{Pe}_T = PrRe \) is the thermal Peclet number, where the Reynolds and Prandtl numbers are defined as \( Re = u_0^* R_0^*/\mu_L^* \) and \( Pr = \mu_L^* C_L^*/k_L^* \), respectively, where the subscript \( L \) denotes the liquid, and \( \mu, k \) and \( C \) are the dynamic viscosity, thermal conductivity and specific heat.
Gases enclosed by the annular liquid jet

\[
\frac{\text{dm}_j}{\text{dt}} = \dot{m}_j + R_M \int_0^L \int_0^{R_1} \omega_j r \, \text{d}r \, \text{d}x, \tag{9}
\]

\[
\frac{\text{d}}{\text{dt}}(m_j T_i) = \gamma \frac{\rho_i^* C_v^*}{\Delta c^* C_p^*} \dot{q}_i - 2(\gamma - 1) \frac{p_i}{p_e} \frac{\text{d}V_j}{\text{d}t} - R_R \sum_{j=1}^N h_j^0 \int_0^L \int_0^{R_1} \omega_j r \, \text{d}r \, \text{d}x, \tag{10}
\]

where

\[
\dot{m}_j = \frac{1}{P e_M} \frac{D_j^* L}{D_L^*} \int_0^L R_1 \frac{\partial \rho_j}{\partial r} (R_1(t,x),x,t)(1 + \tan^2 \theta_1) \, \text{d}x, \tag{11}
\]

\[
\dot{q}_i = \frac{1}{P e_T} \int_0^L R_1 \frac{\partial T}{\partial r} (R_1(t,x),x,t)(1 + \tan^2 \theta_1) \, \text{d}x, \tag{12}
\]

\[
p_{i,j} V_i / p_e = m_j T_i, \quad p_i / p_e = m_j T_i, \tag{13}
\]

\[
m_j(0) = p_i(0) V_i(0) / p_e T_i(0), \tag{14}
\]

\[
p_i = \sum_{j=1}^N p_{i,j}, \quad m_i = \sum_{j=1}^N m_j, \tag{15}
\]

\[
V_i = \int_0^L R_i^2(t,x) \, \text{d}x, \quad \tan \theta_1 = \frac{\partial R_j}{\partial x}, \tag{16}
\]

\[
R_M = \Omega' R_0^* / 2 \Delta c^* u_0^*, \quad R_R = R_M Q' / C_v^* T_e^*, \tag{17}
\]

\( m_j \) is the mass of the \( j \)th gases enclosed by the jet, \( L \) is the convergence length, i.e., the axial distance at which the annular jet merges onto the symmetry axis to become a round one,\( \omega \) is the reaction rate, \( R_M \) is the ratio of a characteristic residence time to a characteristic reaction time, \( Q' / C_v^* T_e^* \) denotes the ratio between the heat of combustion and the sensible internal energy based on the temperature of the gases that surround the annular jet, \( C_p \) and \( C_v \) are the gases’ specific heats at constant pressure and volume, respectively, \( \Omega \) is a reference reaction rate, \( \gamma \) is the specific heat ratio, \( V_i \) is the volume enclosed by the jet, \( N \) is the number of species and \( h_j^0 \) is the enthalpy of formation of the \( j \)th species.

If only one gaseous species is generated by combustion within the volume enclosed by the annular liquid jet, i.e., \( N = 1 \), then one may choose \( C_{eqe}^* = S_e^* p_e^* \), so that the interfacial boundary conditions become
\[ \rho(r, 0, t) = \alpha(\beta - 1), \]  
(18)

\[ \rho(R_1, x, t) = \alpha \left( \delta \frac{p_l}{p_e} - 1 \right), \]  
(19)

\[ \rho(R_2(t, x), x, t) = 0, \]  
(20)

where

\[ \alpha = 2R^* S_e^* T_e^*, \quad \beta = \frac{\rho^*(r, 0, t)}{S_e^* p_e^*}, \quad \delta = \frac{S_i^*}{S_e^*}, \]  
(21)

and we will assume that the concentration of the gases dissolved in the liquid at the nozzle exit is uniform, i.e., \( \rho^*(r, 0, t) = \text{constant} \). The pressure of the gases that surround the annular liquid jet will be assumed to be constant in the remaining of the paper.

**Fluid dynamics of inviscid liquid jets**

For inviscid, slender, annular liquid jets, the nondimensional fluid dynamics equations based on an integral formulation [1] can be written as:

\[ \frac{\partial m}{\partial t} + \frac{\partial}{\partial x} (mu) = 0, \]  
(22)

\[ \frac{\partial}{\partial t} (mu) + \frac{\partial}{\partial x} (muu) = \frac{m}{Fr} + \frac{1}{We} \left( \frac{\partial J}{\partial x} - C_{pn} \frac{\partial R}{\partial x} \right), \]  
(23)

\[ \frac{\partial}{\partial t} (m\overline{v}) + \frac{\partial}{\partial x} (mu\overline{v}) = \frac{1}{We} \left( C_{pn} R - \frac{\partial J}{\partial x} \right), \]  
(24)

\[ \overline{v} = \frac{\partial R}{\partial t} + u \frac{\partial R}{\partial x}, \]  
(25)

\[ J = R \left[ 1 + \left( \frac{\partial R}{\partial x} \right)^2 \right]^{1/2}, \]  
(26)

\[ R_2 = R + b/2, \quad R_1 = R - b/2, \quad b = R_2 - R_1, \]  
(27)

\[ b = \frac{m b_0^*}{R R_0^*}, \]  
(28)

where \( m \) denotes the mass per unit length and per radian of the liquid jet, \( b \) is the jet’s thickness

\[ Fr = u_0^* / g^* R_0^*, \quad We = m_0^* u_0^2 / 2 \sigma^* R_0^*, \]  
(29)

\[ C_{pn} = C_p We, \quad C_p = (p_l^* - p_e^*) R_0^2 / m_0^* u_0^2. \]  
(30)

\( Fr \) and \( We \) are the Froude and Weber numbers, respectively, \( m_0^* = \rho_l^* b_0^* R_0^* \), and \( C_{pn} \) denotes the pressure coefficient. Note that \( C_p \) and \( C_{pn} \) may be functions of \( t \) because \( p_l^* \) may be a function of \( t \).
In the above equations, $u = u(t,x)$ and $\bar{v} = \bar{v}(t,x)$ are the liquid’s average axial and radial velocity components, respectively, $\sigma$ is the surface tension and $R$ is the jet’s mean radius. The radial dependence of the radial velocity component may be obtained by integrating the continuity equation and the result may be written as

$$v(t,r,x) = \frac{1}{r} \left( \frac{\partial}{\partial t} \left( \frac{R^2}{2} \right) + \frac{\partial}{\partial x} \left( \frac{R^2 - r^2}{2} u \right) \right),$$

which is the value to be used when solving for both the gas concentration in and the temperature of the liquid.

The above equations are subject to the following conditions at the nozzle exit

$$m(t,0) = 1, \quad u(t,0) = 1,$$

$$\bar{v}(t,0) = \tan \theta_0, \quad R(t,0) = 1,$$

where $\tan \theta_0 = \partial R(t,0)/\partial x$, and $\theta_0$ is the angle of the jet’s mean radius at the nozzle exit.

The pressure coefficient can also be written as

$$C_{p\infty} = C_{p\max} \left( \frac{p_i^*}{p_e^*} - 1 \right),$$

where

$$C_{p\max} = \frac{p_i^* R_0^*}{2\sigma^*}.$$
3. Presentation of results

The fluid dynamics and heat and mass transfer equations for the liquid summarized in the previous section are nonlinearly coupled in an integrodifferential manner with the energy and mass of the gases enclosed by the annular liquid jet; however, for steady-state conditions, these equations are uncoupled. In order to maintain steady conditions, the heat and mass absorption rates must be equal to those of the gases generated by combustion. If either the mass or heat rate generated by combustion is not equal to its corresponding absorption rate, then the jets are time-dependent on account of either mass or heat accumulation within the volume enclosed by them. In order to maintain steady conditions in the absence of combustion, gases must be injected into the volume enclosed by the jet at rates equal to the heat and mass absorption rates by the liquid.

Since the number of parameters affecting the fluid dynamics and heat and mass transfer processes is very large, we have defined the following basic set of parameters $Re = 10^6$, $Pe_M = 10^6$, $Pe_T = 10^6$, $b(0, t)/R(0, t) = 0.05$, $Fr = 10$, $\theta_0 = 0$, $C_L/C_p = 100$, $\rho_L/\Delta c^* = 1000$, $\gamma = 1.4$, $T_i(0) = 1.5$, $p_i(0)/p_e = 0.75$, $C_{p\text{max}} = \rho^*_L R_0^2/2\sigma^* = 1$, $T(r, 0, t) = 1$, $\alpha = 1$, $\beta = 1$, $\delta = 1$, $N = 1$, $R_M = R_R = 0$, and $We = Wb(0, t)/2R(0, t) = 10$, where $R(0, t) = (R_1(0, t) + R_2(0, t))/2$, $b(0, t) = R_2(0, t) - R_1(0, t)$, $a = 0$, and $\tan \theta_0 = \partial R/\partial x(0, t)$.

In the results presented below, only the effects of g-jitter, i.e., $a$ and $S$, on heat and mass transfer phenomena are considered since the influence of each of the above parameters was studied in Part I [1]. These results were obtained by assuming steady-state conditions and $a = 0$ initially. Since, under steady steady, the fluid dynamics and heat and mass transfer equations are uncoupled, one can first solve for the flow field, and then solve for the heat and mass transfer equations. These steady conditions are referred to as occurring at $t = 0^-$. For $t \geq 0$, mass injection was set to zero, g-jitter was applied, and the dynamic behavior of the jet was determined numerically by means of an adaptive finite difference method that first maps the geometry of the jet into a unit square, and then discretizes the strong conservation-law form of the equations by means of upwind differences for the convection terms, central differences for diffusion and backward differences in time [15]; the resulting system of discrete equations was solved iteratively at each time step until a user’s specified convergence criterion was satisfied. The results presented in this section correspond to $t \geq 0$ where, for the sake of convenience, the temperature at the jet’s inner interface is denoted by $tempi$, the gas concentration at the jet’s inner interface is $ci$, the mass transfer rate at the jet’s inner interface is $dmidt$, the heat fluxes at the jet’s inner and outer interfaces are $dqidt$ and $dqedt$, respectively, and the volume of the gases enclosed by the annular liquid jet is $V_i$ in the figures shown below. The axial velocity profile at the nozzle exit was assumed to be uniform.
Some sample results that illustrate the effects of \( g \)-jitter on the heat and mass transfer processes in annular liquid jets are presented in Figs. 2–9; a more complete study including the jet’s thickness and axial velocity component at the convergence point, the mass flux at the jet’s outer interface and the mass of the gases enclosed by the jet as functions of time can be found in Ref. [16]. Figs. 2 and 3 show that the pressure coefficient, i.e., the pressure of the gases enclosed by the jet, exhibits a periodic behavior after the initial transients have disappeared whose frequency coincides with that of the \( g \)-jitter; however, the pressure coefficient is not in phase with the residual acceleration, and its amplitude increases as \( a \) is increased. A similar behavior has been observed for \( R(t, L(t)) \), \( b(t, L(t)) \) and \( u(t, L(t)) \), although the minima of the jet’s thickness at the convergence point occur nearly at the same time as the maxima of the jet’s mean radius and axial velocity component at the convergence point.

The phase plane for the jet’s mean radius at the convergence point shown in Fig. 2 includes the initial transients, but clearly indicates that once these
transients disappear, this plane exhibits a closed loop whose width and height and, therefore, its area increase as the amplitude of the g-jitter is increased. Fig. 2 also indicates that the centroid of the phase plane moves towards larger mean radii as $a$ is increased. The interfacial gas concentration at the jet’s inner interface exhibits the same trend as those of the pressure coefficient because of its linear dependence on the latter (cf. Eq. (19)).

The mass transfer rate at the jet’s inner interface also exhibits a periodic response whose amplitude increases as the amplitude of the g-jitter is increased, but has a phase lag of about $180^\circ$ with respect to the interfacial gas concentration, i.e., the time at which the maxima of the pressure coefficient are observed nearly coincides with the minima of the mass transfer rate. Although not shown here [16], the mass of the gases enclosed by the jet increases in an oscillatory manner is about 1.6% larger than its initial value at $t \approx 25$, and increases slowly with time because of the small binary diffusion coefficients of gases in liquids [1,15]. The mass transfer rate at the jet’s outer interface is about 54 orders of magnitude smaller than that at the inner interface.
Both the heat flux at the jet’s inner interface and the volume enclosed by the annular liquid jet are periodic functions of time whose amplitude increases as the amplitude of the g-jitter is increased as shown in Fig. 3. The time at which the maxima in the volume enclosed by the jet occur nearly coincide with those at which the pressure coefficient reaches its minima, in accordance with the ideal gas law employed in this study; however, the maxima of the volume enclosed by the jet lags about 90° with respect to the maxima of the heat flux at the jet’s inner interface. The heat flux at the jet’s outer interface is about 50 times smaller than that at the inner one, and the locations of the maxima of the heat fluxes at the jet’s inner and outer interfaces nearly coincide. Note that the nondimensional heat flux at the jet’s inner interface is about 3 orders of magnitude smaller than the mass absorption rate at that interface.

The temperature of the gases enclosed by the jet and, therefore, the interfacial temperature at the jet’s inner interface is periodic but not a sinusoidal
function of time whose amplitude increases as the amplitude of the $g$-jitter is increased; however, the variations in interfacial temperature are small. This is in agreement with the results presented in Part I [1] that indicated that the cooling of annular liquid jets is a faster phenomenon than mass transfer. The interfacial temperature also exhibits a lag of about $60^\circ$ with respect to the pressure coefficient and a lag of about $180^\circ$ with respect to the heat flux at the jet’s inner interface. Moreover, Fig. 3 also shows that heat flux at the jet’s inner interface increases at a larger rate in the compression cycle of the gases enclosed by the jet, because an increase in $p_i$ results in an increase in convergence length, i.e., a larger surface area for heat transfer, and thinning of the jet, i.e., steeper temperature gradients, than in the expansion cycle, whereas the interfacial temperature and the heat flux at the jet’s outer interface decrease.

Similar trends to those shown in Figs. 2 and 3 have also been observed for $S = 0.25$, i.e., the pressure coefficient, interfacial gas concentration and temperature, heat fluxes at the jet’s inner and outer interfaces, mass absorption rate and volume enclosed by the jet exhibit periodic responses for sinusoidal $g$-jitter. The amplitude of these periodic responses increases as the amplitude of
the \( g \)-jitter is increased. For the same value of \( a \), it has been observed [16] that the response amplitude decreases as the frequency of the \( g \)-jitter is increased.

Figs. 4 and 5 illustrate the effects of the Strouhal number on the dynamics of annular liquid jets. Fig. 4 indicates that the pressure coefficient increases slowly as a function of time because of mass transfer from the liquid to the gases enclosed by the jet, and the amplitude of its oscillations decreases as the Strouhal number is increased. A similar behavior is also observed in the jet’s mean radius at the convergence point; the phase diagram shows that the flow tends slowly to a periodic behavior. These results are consistent with the fact that mass transfer in annular liquid jets is a slow phenomenon [1,15,16] as shown in the mass transfer rate at the jet’s inner interface whose mean value decreases as a function of time, whereas the volume and mass of the gases enclosed by the jet increase with time. On the other hand, the interfacial temperature oscillates about its steady value which, of course, coincides with the temperature at the jet’s outer interface. The temperature of the gases
enclosed by the jet is a periodic function of time whose amplitude increases as the Strouhal number is increased; a similar behavior is also observed for the heat flux at the jet’s inner interface. The heat flux at the jet’s outer interface is also a periodic function but its amplitude increases as the Strouhal number is decreased.

The results presented in Figs. 2–5 and others not shown here [16] indicate that mass transfer in annular liquid jets is a slow phenomenon that requires a longer time to reach steady or purely periodic behavior than heat transfer. In any case, these results show that the fluid dynamics of and heat and mass transfer in annular liquid jets respond in a periodic manner to and have the same frequency as that of the applied sinusoidal g-jitter.

In order to verify the slowness of mass transfer in underpressurized, annular liquid jets, the cooling of these jets was investigated numerically for the same set of parameters as those described above in the absence of both combustion and mass transfer, and some sample results are presented in Figs. 6–9 which correspond to the same amplitude and frequency of the g-jitter as Figs. 2–5. Therefore, a comparison between the results presented in Figs. 2–5 and
Figs. 6–9 clearly illustrates the effects of mass transfer when annular jets are forced by gravity modulation.

Figs. 2 and 6 clearly show that the maxima and minima of the pressure coefficient and jet’s mean radius occur at almost the same times when heat and mass transfer and only heat transfer are considered; however, the amplitude of the pressure coefficient is smaller when only heat transfer is considered because, for the underpressurized jets analyzed in this paper, the mass transfer rate is from the liquid to the gases enclosed by the jet. This transfer increases the mass and pressure of the gases enclosed by the jet.

The phase diagram illustrated in Fig. 6 is a closed loop and indicates that for the times considered in that figure the fluid dynamics have reached a purely periodic motion characterized by a single closed curve. Fig. 6 also shows that the frequency of the pressure coefficient and jet’s mean radius at the convergence point is exactly the same as that of the g-jitter.

The heat fluxes, volume enclosed by the jet and interfacial temperature presented in Fig. 7 are almost identical to those of Fig. 3, thus indicating that
heat transfer phenomena in annular liquid jets are fast and the liquid reaches thermal equilibrium long before the mass absorption rate becomes a purely periodic function of time and the pressure coefficient achieves a constant mean value.

Figs. 8 and 9 show that the heat fluxes at the jet’s inner (bottom left) and outer (bottom right) interfaces, volume enclosed by the jet and interfacial temperature are all periodic functions of time with constant mean values. The amplitude of the interfacial temperature and heat flux at the inner interface decreases whereas those of the volume enclosed by the jet and heat flux at the outer interface increase as the Strouhal number is increased. Figs. 4 and 5 and Figs. 8 and 9 also show the response of the annular jet when there is no $g$-jitter. These figures clearly indicate that the jet has reached a steady-state condition when there is no $g$-jitter; in the presence of mass transfer, the mass, volume and pressure of the gases enclosed by the jet increase slowly with time until reaching a steady value because the gases are transferred from the liquid to the volume enclosed by the jet, thereby increasing the pressure of the gases enclosed by the jet which, in turn, results in an increase in volume.
Figs. 2–9 and other results not presented here [16] indicate that, for the amplitudes and frequencies of the g-jitter considered here, the nonlinear, integrodifferential coupling between the fluid dynamics and heat and mass transfer equations do not result in heat or mass transfer enhancement. This result may be a consequence of the fact that the convergence length, surface area of the jet and volume enclosed by the jet are not strong functions of the Froude number.

4. Conclusions

Heat and mass transfer in underpressurized, annular liquid jets has been investigated as a function of the amplitude and frequency of the g-jitter. It has been found that the pressure coefficient, the jet’s mean radius, thickness and axial velocity component at the convergence point, gas concentration at the jet’s inner interface, heat fluxes at the jet’s inner and outer interfaces, volume enclosed by the jet and interfacial temperature are periodic functions of time whose amplitudes increase as the amplitude of the g-jitter is increased, but decrease as the jitter frequency is increased. The pressure coefficient is almost in phase with the jet’s thickness at the convergence point and the heat flux at the jet’s outer interface, and out of phase with the jet’s mean radius and axial velocity component at the convergence point, mass transfer rate at the jet’s inner interface and volume enclosed by the jet.

Despite the nonlinear, integrodifferential coupling between the fluid dynamics and heat and mass transfer equations, the annular jet’s response is periodic and depends almost linearly on the amplitude of the gravity modulation as a consequence of the small dependence of the convergence length and surface area of and volume enclosed by the jet on the Froude number.

It has also been found that the energy or temperature field adapts itself rapidly to the imposed gravity modulation, i.e., heat transfer phenomena are fast, and thermal equilibrium is reached quickly. However, the small binary diffusion coefficient of gases in liquids implies small mass transfer rates, and underpressurized jets subject to g-jitter grow with time until a steady state is reached. Since the ratio between the characteristic mass and heat diffusion times is at least one hundred, calculations must be performed for much longer times than the ones presented here in order to observe that, once the mass absorption rate is nil, the interfacial gas concentration, mass transfer rate at the jet’s inner interface and mass and volume of the gases enclosed by the jet are periodic functions of and have the same frequency as that of the g-jitter.
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