On centralized power pool auction: a novel multipliers stabilization procedure

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Abstract

This paper addresses the Short-Term Hydro-Thermal Coordination (STHTC) problem. It is a large-scale, combinatorial and nonlinear optimization problem. It is usually solved using a Lagrangian Relaxation (LR) approach. LR procedure is based on the solution of the dual problem of the original one. The dual problem variables are the Lagrange multipliers. These multipliers have an economic meaning: electric energy hourly prices. This paper focuses in an efficient solution of the dual problem of the STHTC problem. A novel multiplier stabilization technique, which significantly improves the quality of the solution, is presented. The provided method could be the optimization tool used by the Independent System Operator of a centralized Power Pool. The solution procedure diminishes the conflict of interest in determining energy prices. A realistic large-scale case study illustrates the behavior of the presented approach.

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1. Introduction

Liberalized electricity energy systems have replaced the centralized operational and control functions of the traditional systems by different markets that trade energy and services in a competitive manner. There is a great diversity of markets designs; practically, not two equal market designs exist. Wilson [1] identifies three groups of market models: (i) centralized models (such as the PJM Power Pool and the New York Power Pool), (ii) decentralized models (such as the Californian and Spanish markets), and (iii) hybrid models (such as the New Zealand market). Centralized models are characterized by the creation of an Independent System Operator that executes a central cost-minimization scheduling of generation. These models use the optimization theory developed to solve the classic Short-Term Hydro-Thermal Coordination (STHTC) or Unit Commitment (UC) problems.

This paper addresses the STHTC problem. This problem is solved in the traditional non-competitive electric energy systems to determine the start-up and shut-down schedule of thermal plants, as well as the power output of thermal and hydro plants during a short-term planning horizon. The target is to meet customer demand with appropriate levels of spinning reserve such that total operating costs are minimized.

This is a large-scale mixed-integer nonlinear problem. As recognized in the technical literature, the Lagrangian Relaxation (LR) technique is the most promising procedure to solve the STHTC problem [2–11]. The LR technique has also been used to solve scheduling problems in deregulated systems [12,13]. Other solution techniques to solve the STHTC problem are: Dynamic Programming [14], Mixed Integer Linear Programming [15–17] and Interior Point Methods [18]. In [18] the STHTC problem is solved using a combination of Genetic Algorithms and Interior Point Methods. A Genetic Algorithm is used to compute the optimal commitment variables while an Interior Point Method is used to solve the Hydro-Thermal Economic Dispatch.

The LR procedure is based on the solution of a dual problem (DP) of the original one, called primal problem (PP). The variables of the DP are the Lagrange multipliers. Because the STHTC problem is non-convex [19],
the solution of the DP is not the solution of the PP but it is a lower bound. The relative difference between the optimal cost values for the primal and dual problems is the duality gap. To measure the quality of a solution found by a LR algorithm, an upper bound of the duality gap can easily be computed using the best PP and DP costs.

Once the solution of the DP is found, and using heuristic procedures, a solution to the PP is easily obtained. The LR technique to solve the STHTC consists of the three phases below [2–6,8–10]:

Phase 1. Solving the DP.
Phase 2. Obtaining a feasible set of commitment variables.
Phase 3. Exactly dispatching committed generation to meet the demand.

In Power Pool auctions based on optimization models [1,20], the solution of Phase 1 may raise equity concerns [21,22]. This paper focuses on Phase 1. This is the most complex stage of the algorithm and the one that determines the quality of the final solution. Phase 1 has usually been solved through subgradient techniques [2,3,5,6]. More efficient techniques have recently been reported: Bundle Methods [7,8,10], Dynamically Constrained Cutting Plane Methods [9] and Interior Point Methods [11].

In Phase 2, the solution of the dual problem is slightly modified to achieve a feasible set of commitment variables [2,5,9]. Phase 3 is a multi-period economic dispatch whose solution is well stated in the technical literature [23].

The dual function of the STHTC problem is concave [19] and is usually flat near the optimum. This means that, although the values of the multipliers may vary by a significant amount, the change in the value of the objective function is almost negligible. For this reason, the stopping criterion for Phase 1 is typically the stabilization of the DP cost or the execution of a fixed number of iterations [2–11]. Achieving a tight convergence criterion in the Lagrange multipliers has a high computational cost, which is not justified, in the framework of non-competitive electric energy systems, by the small increase in the DP cost and the potential decrease in the PP cost. However, in a centralized competitive electric Power Pool, the DP procedure of solution should attain convergence in the DP cost and in the Lagrange multipliers, which represent electric energy prices. If the Lagrange multipliers have not converged (or have not been stabilized) before the end of the procedure, some units are privileged versus the rest [24].

This paper presents a method to solve the DP of the STHTC problem. Alike previously proposed approaches, the procedure achieves convergence in the DP cost and in the Lagrange multipliers. This improvement in the DP convergence approach notably reduces the upper bound of the duality gap. In [25], it is shown that the cost not recovered by Lagrange multipliers, if used as prices in a centralized Power Pool, is bounded above by the duality gap. Because Lagrange multipliers are stabilized in the proposed method, they become appropriate signals to derive prices and the conflict of interest in determining energy prices is diminished.

The proposed STHTC approach could be used in a centralized electricity market such as the PJM or the NY Power Pools. Furthermore, this procedure could also be the hydro-thermal coordination algorithm used for a generation company to plan its unit’s production to meet its share of demand and to bid in the daily electric energy market [26,27].

This paper is organized as follows. Section 2 formulates the STHTC problem. Section 3 introduces a method to solve the DP achieving convergence not only on the DP cost, as other procedures in the literature, but also on the Lagrange multipliers. This feature makes the method a useful tool in a competitive environment. Section 4 includes a large-scale case study to illustrate the behavior of the algorithm. Section 5 provides conclusions.

2. Problem formulation

The STHTC problem can be formulated as follows:

\[
\text{Minimize}_{x,y} \quad f(x) = \sum_i f_i(x_i) \tag{1}
\]

subject to

\[
s_i(x_i) \leq 0 \quad \forall i \tag{2}
\]

\[
s_j(y_j) \leq 0 \quad \forall j \tag{3}
\]

\[
h(x,y) = H - \sum_i h_i(x_i) - \sum_j h_j(y_j) = 0 \tag{4}
\]

\[
g(x,y) = G - \sum_i g_i(x_i) - \sum_j g_j(y_j) \leq 0, \tag{5}
\]

where \((x,y) = (x_i,y_j; \forall i, \forall j)\), variables \(x_i\) are the variables related to thermal plant \(i\), and \(y_j\) are the variables related to the hydroelectric system \(j\). \(G, g_i(x_i), g_j(y_j), H, h_i(x_i)\) and \(h_j(y_j)\) are vectors of dimension equal to the number of subperiods in the planning horizon. It should be noted that time is embedded in the above formulation.

Eq. (1) is the production cost to be minimized. The total production cost is the addition of start up and shut down costs and operation costs of every thermal unit in the system.

Eq. (2) model the constraints related to every thermal unit. Thermal constraints include: minimum and maximum unit operating limits, maximum down and up ramp rates, minimum up time and minimum down time of every thermal unit.

Eq. (3) represent the constraints related to every hydroelectric system. Hydro constraints include: water balance equations, upper and lower bounds on turbine
discharges and reservoir volumes, upper bounds on water spillages and initial and final conditions on reservoir volumes. The power produced by a hydro unit is modeled as a nonlinear function of the water discharge and the head of the reservoir. Nevertheless, the head variations are rather small and can be neglected for the short-term planning period.

Eqs. (4) and (5) are the load (or global) constraints. They couple thermal-related and hydro-related variables. Eq. (4) includes global equality constraints, i.e. a demand constraint for each subperiod of the time horizon. Eq. (5) includes global inequality constraints. Typically, only spinning reserve constraints are considered. Problem (1)–(5) is named the primal problem (PP).

The above formulation was presented in [9]. Network constraints are not considered. Applying the LR method, a vector of multipliers \( \lambda \) is associated with the vector of Eq. (4) and a vector of multipliers \( \mu \) is associated with the vector of constraints (5). The derivation of the Lagrangian function, the dual function \( \phi(\lambda, \mu) \) and the decomposed primal problem (DPP) for the problem above (1)–(5) can be found in [9]. The DPP is the optimization problem required to evaluate the dual function for each given value of the multiplier vector \( \theta = (\lambda, \mu) \). The DPP is decomposed into one subproblem per thermal unit and one subproblem per hydroelectric system. The formulation of the subproblem associated with each thermal unit and each hydroelectric system can be found in [9,28]. It will be repeated here to emphasize the economical meaning of the multipliers [29].

The subproblem associated with thermal unit \( i \) is:

\[
\begin{align*}
\text{Minimize}_x & \quad f_i(x_i) - \left[ \lambda_i^T h_i(x_i) + \mu_i^T g_i(x_i) \right] \\
\text{subject to} & \quad s_i(x_i) \leq 0.
\end{align*}
\]

The objective function (6) of the above problem includes two terms. The first term represents the cost of the unit to commit power and produce energy in each subperiod of the time horizon considered. The second term represents the unit revenues for its contribution to the system load constraints. If the load constraints are the demand and the spinning reserve constraints in each subperiod \( k \), then each component \( \lambda_k \) of the multiplier vector \( \lambda \) represents the revenue for each MWh of energy produced in subperiod \( k \). Similarly, each component \( \mu_k \) of the multiplier vector \( \mu \) represents the revenue for each MW of power contributing to meet the spinning reserve constraint in subperiod \( k \). Therefore, each thermal unit solves an optimization problem where total costs are minimized (or equivalently, total profit is maximized).

The subproblem associated with hydroelectric system \( j \) is:

\[
\begin{align*}
\text{Maximize}_y & \quad \lambda_j^T h_j(y_j) + \mu_j^T g_j(y_j) \\
\text{subject to} & \quad s_j(y_j) \leq 0.
\end{align*}
\]

The objective function (8) of the above subproblem represents total revenues for all the units in the same hydroelectric system. Hydropower production costs are neglected. The economic meaning of each \( \lambda_k \) and \( \mu_k \) multipliers is the same as above. Again, each hydroelectric system solves an optimization problem to maximize its profit.

The dual problem (DP) of the original primal problem (1)–(5) has the form:

\[
\begin{align*}
\text{Maximize}_{\lambda, \mu} & \quad \phi(\lambda, \mu) \\
\text{subject to} & \quad \mu \geq 0.
\end{align*}
\]

The variables of the DP are the Lagrange multipliers, which have the economic meaning stated above. The Lagrangian Relaxation procedure to solve the dual problem works as follows [9]:

1. Initialize multiplier vector \( \theta = (\lambda, \mu) \).
2. Solve the decomposed primal problem by solving one subproblem per thermal plant (6)–(7) and one subproblem per hydroelectric system (8)–(9).
3. Update the multiplier vector \( \theta \).
4. If the convergence criteria are met, stop. Otherwise go to 2.

3. Solution procedure

This paper presents an efficient procedure to solve the DP of the STHTC problem. This approach achieves convergence in the DP cost and in the Lagrange multipliers.

The proposed method solves the DP in two consecutive stages. A different multiplier updating procedure is used in each stage. In the first one, a near optimal DP solution is obtained. The stopping criterion, for this stage, is the convergence of the DP cost function. In the second one, the solution of the first stage is refined and the multipliers are stabilized. The procedure is described below:

**Phase 1**

**Stage A** The DP is solved using the Dynamically Constrained Cutting Plane Method (DC-CP) [9]. At the end of this stage the DP cost has converged.

**Stage B** The solution of the DP is improved starting from the solution of Stage A and using subgradient techniques to update the multipliers.

**Phase 2** Heuristic procedures are applied to achieve a feasible set of commitment variables.

**Phase 3** Committed units are optimally dispatched to meet the demand.

In the first stage of Phase 1, the DP is solved using the DC-CP method. The DC-CP method achieves convergence of the DP cost in a fast and efficient way. This method computes, in each iteration, an upper bound and a lower
bound of the DP optimum. When the difference between these bounds is small enough, the process is stopped. In [9], the DC-CP method is compared to other methods in the literature: subgradient method [2,3,5,6,30], cutting plane method [30], and bundle methods [7,10,30]. It outperforms these other methods in terms of the quality of the solution and the required CPU time.

Even though, at the end of the first stage of Phase 1 the DP cost has converged, the multipliers still present an oscillating behavior. In the second stage of Phase 1, the solution obtained within the first stage is upgraded using subgradient techniques and the multipliers are stabilized. The subgradient method updates the multipliers proportionally to the mismatches in the load constraints. It is well known that this method is not computationally efficient. However, if a good starting point is available the subgradient method could be a competitive alternative improving the quality of the DP solution. Moreover, as the number of iterations in the second stage of Phase 1 increases, the mismatches in the demand and spinning reserve constraints decrease and, as a consequence, the solution of the second stage (Stage B) is closer to the feasibility of the primal problem than the solution of first stage (Stage A).

Solving the DP in two consecutive stages has three important benefits: (i) the upper bound of the duality gap is reduced; (ii) the Lagrange multipliers are stabilized in an efficient way; and (iii) the solution that Stage B yields is closer to a primal feasible solution as compared to the solution obtained in Stage A. Consequently, the heuristic procedure to achieve a primal feasible solution starting from the solution of the dual problem (Phase 2) requires less CPU time when a multiplier stabilization stage (Stage B of Phase 1) has been added to the DP solution procedure. The small increment in CPU time due to the introduction of the multipliers stabilization stage can be compensated by the reduction in the CPU time required to achieve a primal feasible solution (Phase 2).

4. Case studies

To illustrate the properties of the proposed procedure, two case studies are analyzed. Both are realistic large-scale case studies based on the electric energy system of mainland Spain. The generating system consists of 70 thermal plants and 30 hydraulic plants in one complex hydroelectric system. The planning horizon is one day hourly divided. The reported CPU time refers to a Silicon Graphic Workstation with a R10000 processor and 192 MB of RAM. The model was developed in FORTRAN 77. The commercial optimizer MINOS [31] was used to solve hydro subproblems (Eqs. (8)–(9)) and the economic dispatch procedure of Phase 3. Thermal subproblems (Eqs. (6)–(7)) were solved by dynamic programming techniques.

Total primal problem cost is the sum of production cost, start-up cost and shut-down cost for all thermal units in all subperiods. The production cost of every thermal plant is considered as a quadratic function of the output power. The start-up and shut-down costs of every thermal plant are considered constant. Ramp rate limits, minimum up time and minimum down time constraints are enforced. Hydropower of each hydroelectric unit is modeled as a piecewise linear function of water discharge. Regarding load constraints, demand and spinning reserve constraints are included. The required spinning reserve for each hour is greater than or equal to 10% of the demand in that hour.

Both case studies correspond to the solution of the STHTC problem by LR techniques. In the first one, the algorithm proposed in this paper is used. This case study will be denominated Case 1. It solves the DP (Phase 1) in two successive stages. In the second case study, which will be called Case 2, the DP is solved in only one stage using the DC-CP method [9]. Tables 1–4 present results corresponding to both case studies. Figs. 1–7 refer to Case 1.

Tables 1 and 2 show, respectively, the number of iterations and the CPU time required in each phase of cases 1 and 2. It happens that the number of iterations and the CPU time required to achieve convergence in Phase 2 of
Case 1 are significantly lower than the corresponding amounts for Case 2. Consequently, the introduction of a multiplier stabilization stage (Stage B of Phase 1) significantly reduces the number of iterations and thus the CPU time required to achieve a feasible set of commitment variables. This is so because the solution of Phase 1 in Case 1, is much closer to a primal feasible solution than the solution of Phase 1 in Case 2. By comparing the CPU time required for each phase of both cases studies, it can be concluded that the increase in the CPU time due to the introduction of a multiplier stabilization stage in Phase 1 is partly compensated by the decrease in the CPU time required to achieve a feasible set of commitment variables (Phase 2). For these particular case studies, the difference between the sum of the CPU times required by Phase 1 and Phase 2 for both case studies, is only 0.6 s. For this reason, the difference of the total CPU time is almost negligible (see Table 4).

Table 3 shows that DP and PP costs are better in Case 1 than in Case 2. The introduction of a multiplier stabilization stage (Case 1) causes, with respect to Case 2, an increase in the dual function cost of 0.187% and a decrease in the primal function cost of 0.199%. As a consequence, the upper bound of the duality gap is notably reduced, from 0.409% (Case 2) to 0.0214% (Case 1).

Therefore, from the analysis of the figures in Tables 1–4, it can be concluded that the introduction of a multiplier stabilization stage causes an improvement of global problem results: much smaller upper bound of duality gap, higher DP cost and lower PP cost. These results prove the significant improvement in the quality of the obtained solution. Besides, the increment of the total CPU time due to the introduction of Stage B in Phase 1 is negligible.

Some results corresponding to Case 1 are presented below. Figs. 1 and 2 include hourly electric energy demand

![Fig. 1. Demand curve and λ multipliers curve (multiplied by 8) at the end of Stage A of Phase 1 (Case 1).](image1)

![Fig. 2. Demand curve and λ multipliers curve (multiplied by 8) at the end of Stage B of Phase 1 (Case 1).](image2)

![Fig. 3. μ multipliers at the end of Stage A (a) and Stage B (b) of Phase 1 (Case 1).](image3)

![Fig. 4. Excess of committed and produced power at the end of Stage A of Phase 1 (Case 1).](image4)
and the vector of multipliers $\lambda$ at the end of Stage A (Fig. 1) and Stage B (Fig. 2). For comparison purposes, the vector of multipliers $\lambda$ has been scaled 8 times greater.

The demand curve presents two peaks. The first one occurs in hours 12 and 13, being the demand in the latter hour slightly greater than in the former. The second peak corresponds to hour 18. Demand in hour 18 is a bit lower than demand in hour 12. Therefore, the three hours of top demand are (from larger to lower values) 13, 12 and 18. The demand curve presents a minimum in hour 5.

Fig. 3 includes the $m$ multiplier vector at the end of Stage A (Fig. 3a) and Stage B (Fig. 3b). As it is the usual case for realistic size STHTC problems, $m$ multipliers are equal to zero in most of the subperiods.

It happens (Fig. 1) that increases and decreases in the hourly curve of $\lambda$ multipliers at the end of Stage A are not proportional to the variations in the hourly demand curve. At the end of Stage A, the maximum value of the vector of multipliers $\lambda$ is not associated with the hour of peak demand. The minimum value of this vector does not occur at the minimum demand hour either. This little correspondence between the shapes of the $\lambda$ multiplier curve at the end of Stage A and the demand curve is due to the oscillating behavior of the Lagrange multipliers, even though the DP cost has converged.

However, at the end of Stage B the shape of the $\lambda$ multiplier curve is matched to the shape of the demand curve (Fig. 2). Though the maximum value of the vector of multipliers $\lambda$ occurs in hour 12 (second higher peak of
With respect to the \( \mu \) multiplier curve at the end of Stage B, the maximum value occurs in the hour of maximum demand (hour 13). Actually, the high value of \( \mu_{13} \) justifies the fact that \( \lambda_{12} \) is slightly higher than \( \lambda_{13} \). The second maximum value of the vector of multipliers \( \mu \) is associated with the second peak of the demand curve (hour 18). Therefore, from the analysis of Figs. 1–3, it can be stated that at the end of Stage A the shapes of the multiplier curves and the demand curve are related. This is not the case at the end of Stage B. Due to the flat shape of the dual function near the optimum, the Lagrange multipliers have not been stabilized at the end of Stage A even though the DP cost has converged. In addition, the difference between the maximum and minimum values of both \( \lambda \) and \( \mu \) multipliers is lower at the end of Stage B than at the end of Stage A. Note that the lowest values in the vertical axis of Figs. 1 and 2 are different and the ranges of variation in the vertical axis of Fig. 3a and b are different too.

Figs. 4 and 5 include the excess of committed power and the excess of produced power in every hour of the planning horizon at the end of Stage A (Fig. 4) and at the end of Stage B (Fig. 5). It is observed than the solution at the end of Stage B is closer to the primal problem feasibility than the solution at the end of Stage A. This is, once again, a consequence of the convergence of the multipliers. Note that at the end of Stage B, there are very few subperiods not meeting the spinning reserve constraint and that the produced power is close to the demand power in many subperiods. Again, the vertical axis ranges of Figs. 4 and 5 are different.

Fig. 6 presents the evolution of two \( \lambda \) multipliers: \( \lambda_{16} \) (Fig. 6a) and \( \lambda_{23} \) (Fig. 6b), while Fig. 7 presents the evolution of two \( \mu \) multipliers: \( \mu_{19} \) and \( \mu_{20} \). For each one of these multipliers, two plots are included. The upper plot shows the evolution of the multiplier through stages A and B of Phase 1 (in the case of \( \lambda \) multipliers, i.e. Fig. 6) or through Phase 1 (stages A and B) and Phase 2 (in the case of \( \mu \) multipliers, i.e. Fig. 7). The lower plot represents, in a smaller vertical axis scale, only the evolution through Stage B of Phase 1 (in the case of \( \lambda \) multipliers, i.e. Fig. 6) or through Stage B of Phase 1 and Phase 2 (in the case of \( \mu \) multipliers, i.e. Fig. 7). Dashed lines mark the border between stages A and B and between Stage B and Phase 2. It is observed that at the end of Stage A the multipliers still present an oscillatory behavior despite of the convergence of the dual function.

In summary, Stage B of Phase 1 in the proposed algorithm achieves the stabilization of the multipliers in an efficient way. The increment in the CPU time (with respect to a process not including this multiplier stabilization stage) is almost negligible. Because the multipliers have converged, the economic meaning they provide is reliable.

5. Conclusions

This paper presents a novel solution procedure to solve the DP of the STHTC problem. The proposed approach solves the DP in two consecutive stages. It achieves convergence in the DP cost and in the Lagrange multipliers. The provided method could be used as the optimization tool used by the Independent System Operator of a centralized Power Pool. Because the Lagrange multipliers are stabilized, the solution procedure diminishes the conflict of interest in determining energy prices.

Extensive computational results, based on the electric energy system of mainland Spain, are presented. In these case studies, it is shown that the presented method significantly reduces the upper bound of the duality gap and that the multipliers are efficiently stabilized.

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References

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