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An Alternative Method to Determine the Magic Tyre Model Parameters Using Genetic Algorithms

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SUMMARY

Tyre behavior plays an important role in vehicle dynamics research. Knowledge of tyre properties is necessary to properly design vehicle components and advance control system. For that purpose mathematical models of the tyre are being used in vehicle simulation models. The Magic Formula Tyre Model is a semi-empirical tyre model which describes tyre behavior quite accurately. The Magic Formula Tyre Model needs a set of parameters to describe the tyre properties; the determination of these parameters is dealt with in this paper. A new method based on genetic techniques is used to determine these parameters. The main advantages of the method are its simplicity of implementation and its fast convergence to optimal solution, with no need of deep knowledge of the searching space. So to start the search, it is not necessary to know a set of starting values of the Magic Formula parameters. The comparison between analytical optimization methods and the method proposed is discussed in this paper.

1. INTRODUCTION

Tyre models are used to calculate the tyre forces and moments as responses to the wheel motion that may be given in terms of various slip quantities. We can distinguish theoretical models based on physics of the tyre construction [1, 2], and empirical or semi-empirical models which are solely based on experimental results [3, 4]. Also, combinations of both approaches are used in the development of the tyre model.

A widely used empirical tyre model is based on the so-called Magic Formula [3]. The development of the model was done in a cooperative effort by the Delft University of Technology and Volvo Car Corporation. This model is capable of describing steady-state tyre force and moment characteristics at pure and at combined

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slip conditions. Michelin introduced a purely empirical method using Magic Formula based functions to describe the tyre force generation at combined slip [4]. These tyre models use tyre measurement data as a basis. A set of Magic Formula parameters derive from tyre measurement data. One procedure to find these parameters is described in [5]. In this approach a regression method to the measurement data in order to derive the parameters of the Magic Formula is used. A main characteristic of this technique is that it requires starting values for the parameters to begin the optimization process. In case of pure slip conditions most of the parameters of the Magic Formula have a physical meaning, so a good approximation of starting values can be made, but good starting values for the combined slip conditions may be difficult and this is a limitation for the time needed to calculate the suitable parameters and to be successful in getting a global minimum.

The approach presented in this paper to determine the Magic Formula parameters deals with genetic algorithms. Genetic algorithms were firstly introduced by Holland [6, 7], whose work is included in Goldberg [8] and they have been extensively and successfully applied to different optimization problems. These methods define a starting population that is improved by approximations to the goal function making use of mechanisms of natural selection and laws of natural genetics. The main advantages of these methods are their simplicity in implementing the algorithms and their low computational cost. Moreover, there is no need to have a deep knowledge of: neither searching space nor the presence of local minima nor other mathematical characteristics required for traditional searching algorithms as well as of the goal function continuity. Also a good characteristic of the method presented is that it does not require starting values to begin the optimization process. This is an important feature because the user of this method does not necessarily have to know the physical characteristic of the Magic Formula parameters.

2. OPTIMIZATION METHOD

Genetic algorithms are different from more normal optimization and search procedures in four ways:

- Genetic algorithms work with a coding of the parameter set, not the parameter themselves.
- Genetic algorithms search from a population of points, not a single point.
- Genetic algorithms use evaluation of goal function, not derivatives or other auxiliary knowledge.
- Genetic algorithms use probabilistic transition rules, not deterministic rules.

Taken together, these four differences contribute to a genetic algorithm's robustness and resulting advantage over other more commonly used techniques.

The optimization problem is given by:

$$\begin{aligned} &\min \quad f(p_1(X), p_2(X), \ldots, p_n(X)) \\ &\text{subject to:} \\ &g_j(X) \leqslant 0 \quad j = 0, 1, 2, \ldots, m \\ &x_i \in [li_i, ls_i] \quad \forall x_i \in X \end{aligned} \tag{1}$$

Where *f* is the goal function, which will be dealt with in the next section, where each individual *X* obtains a value, its fitness, $p_i(\cdot)$ are functions of the properties that show the objectives of the system to be optimized and $g_j(\cdot)$ are the constraints defining the searching space.

The strategy of evolutionary methods for optimization problems begins with generation of a starting population. For the problem dealt with the starting population is a set of parameters of the Magic Formula, whose values are randomly generated within the searching space. Each individual (chromosome) of the population is a possible solution to the problem and it is formed by parameters (genes) that set the variables of the problem.

Genes can be schematized by several forms. In the first approach by Holland [6, 7] they are binary chains, so each gene x_i is expressed by a binary code of size *p*. Therefore precision depends on *p*, and the genes are confined to a range [li, ls], defined either by integer or real values.

Another way to express the genes, as followed in this paper, is directly as real values. All genes are grouped in a vector that represents a chromosome, Storn and Price [9], Wright [10]:

$$X = [x_1 x_2 \ldots x_n] \quad \forall x \in \Re \tag{2}$$

Next the starting population has to evolve to populations where individuals are a better solution. This task can be reached by natural selection, reproduction, mutation or other genetic operators. In this work selection and reproduction are carried out sequentially and mutation is used as an independent process.

2.1. Selection

For selection, two individuals are randomly chosen from the population and they form a couple for reproduction. The selection can be based on different probability distributions, as the uniform distribution or a random selection from a population where the weight of each individual depends on its fitness, so that the best individual has the greatest probability to be chosen.

In this paper, the best individual and two individuals randomly selected with uniform distribution are chosen for reproduction and they make up a disturbing vector, *V*. The scheme, Storn and Price [9], known as differential evolution yields:

$$\begin{aligned} X_i &: i \in [1, NP] \\ V &= X_{best} + F \cdot (X_{r1} - X_{r2}) \end{aligned} \tag{3}$$

where X_{best} is the best individual of a population of *NP* individuals, X_{r1} and X_{r2} are two individuals randomly selected in the population, and *F* is a real value that controls the disturbance of the best individual.

2.2. Reproduction

Next, for reproduction, *V* is crossed with the individual *i* of the current population to generate the individual *i* of the next population. This operator is named crossover.

In natural reproduction, parents' genes are interchanged to form the genes of their descendant or descendants. Reproduction is approached in two ways, on one hand, as shown in Figure 1, parents, X_i and *V*, reproduce and generate their descendant, ${X_N}^i$, by piecewise multipoint crossover: the crossing points, $(j,\ j+k)$, of the segments of each parent's gene for its descendant are randomly selected with uniform probability distribution, so they can be located over the entire genes.

On the other hand, as shown in Figure 2, a discrete multipoint crossover can be used to generate ${X_N}^i$: parent X_i provides its descendant with a set of *N* genes randomly chosen over its entire chromosome and parent *V* provides the rest.

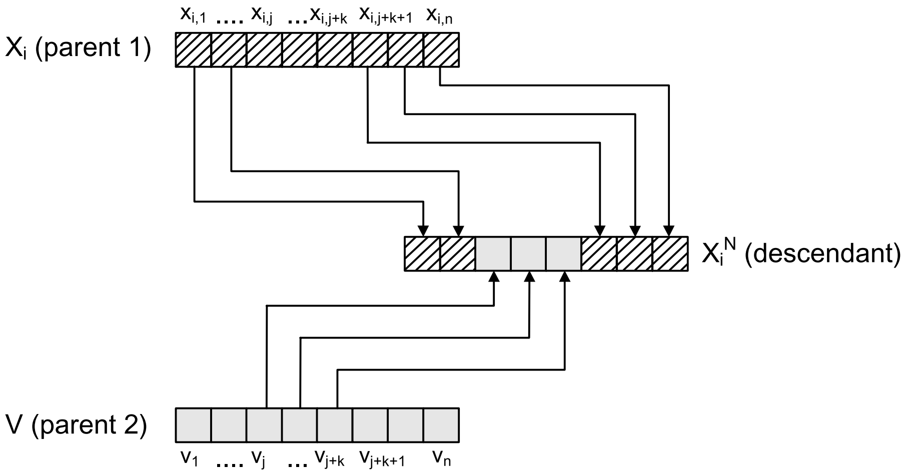

Fig. 1. Piecewise multipoint crossover for reproduction.

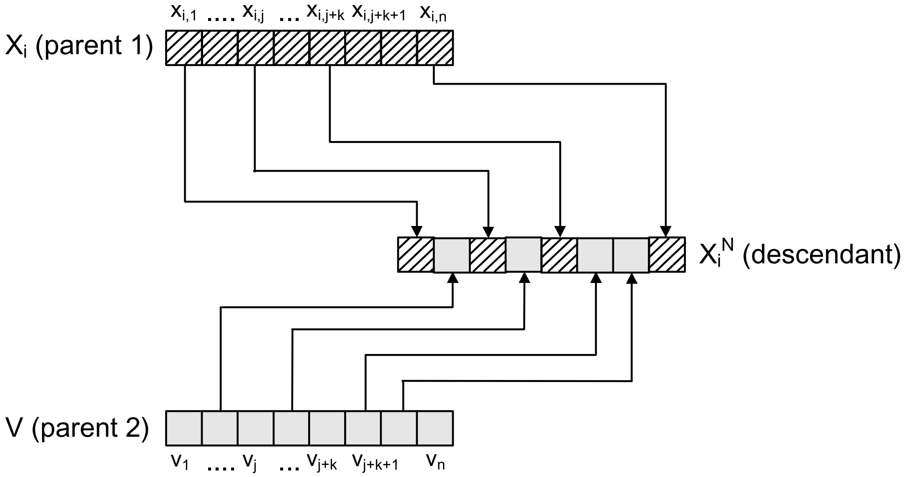

Fig. 2. Discrete multipoint crossover for reproduction.

If the new descendant, ${X_N}^i$, is better than its antecedent, X_i, will replace it. Otherwise X_i is retained and ${X_N}^i$ is rejected. Therefore the population neither increases nor decreases.

Crossover is carried out with a probability defined as CP $\in [0, 1]$.

2.3. Mutation

Mutation is an operator consisting of random change of a gene during reproduction. In this work, mutation is defined as follows: when gene x_i mutates, the operator randomly chooses a value within the interval of real values $(\mathrm{x_i}, \mathrm{x_i} \pm \mathrm{range})$, which is added or subtracted from x_i, depending on the direction of the mutation.

Mutation is carried out with a probability defined as MP $\in [0, 1]$, much lower than *CP*.

3. GOAL FUNCTION

For this particular example the goal function will be obtained comparing the Magic Formula results with the test data. The optimization method will refer to pure and combined slip conditions. The Magic Formula [3] at pure slip conditions is expressed as:

$$y(x) = D \cdot \sin\left[C \cdot \arctan\{B \cdot x - E \cdot (B \cdot x - \arctan(B \cdot x))\}\right] \tag{4}$$

where:

$$Y_{pure}(X) = y(x) + S_v \qquad x = X + S_h \tag{5}$$

Where the output variable Y_{pure} represents the braking and traction force F_x when the input variable X represents the longitudinal slip κ and Y_{pure} represents the lateral force F_y and the auto aligning torque M_z when the input variable X represents the slip angle α. The influence of a tyre's camber angle on its characteristics is ignored, since the characteristics of the tyre under examination are measured at zero camber. The coefficients for each tyre characteristic are expressed as follows and their meaning is presented at Pacejka [3]:

Longitudinal force (pure longitudinal slip):

$$\begin{aligned}
D &= b_1 \cdot F_z^2 + b_2 \cdot F_z \\
C &= b_0 \\
B \cdot C \cdot D &= (b_3 \cdot F_z^2 + b_4 \cdot F_z) \cdot e^{-b_5 \cdot F_z} \\
B &= B \cdot C \cdot D / C \cdot D \\
E &= (b_6 \cdot F_z^2 + b_7 \cdot F_z + b_8) \cdot (1 - b_{13} \cdot \mathrm{sgn}(\kappa + S_{hx})) \\
S_{hx} &= b_9 \cdot F_z + b_{10} \\
S_{vx} &= b_{11} \cdot F_z + b_{12}
\end{aligned} \tag{6}$$

Lateral force (pure lateral slip):

$$\begin{aligned}
D &= a_1 \cdot F_z^2 + a_2 \cdot F_z \\
C &= a_0 \\
B \cdot C \cdot D &= a_3 \cdot \sin\left(2 \cdot \arctan\left(\frac{F_z}{a_4}\right)\right) \\
B &= B \cdot C \cdot D / C \cdot D \\
E &= (a_6 \cdot F_z + a_7) \cdot (1 - a_{17} \cdot \mathrm{sgn}(\alpha + S_{hy})) \\
S_{hy} &= a_8 \cdot F_z + a_9 \\
S_{vy} &= a_{11} \cdot F_z + a_{12}
\end{aligned} \tag{7}$$

In this paper the coefficients for auto aligning torque were not obtained, but the obtaining procedure is the same. For combined slip conditions, the Delft Tyre 96' version is used. This method was developed by Michelin and published in [4]. It describes the effect of combined slip on both the lateral and the longitudinal forces. Weighting functions G are introduced which when multiplied with original pure slip functions produce the combined effects of κ on F_y and of α on F_x, so the Magic Formula at combined slip conditions is expressed as:

$$Y_{combined}(X) = G(x) \cdot Y_{pure}(X) + S_V \tag{8}$$

where the coefficients for pure longitudinal characteristic are presented at Pacejka [11] as:

$$\begin{aligned}
D &= (PDX1 + PDX2 \cdot df_z) \cdot F_z \\
C &= PCX1 \\
E &= (PEX1 + PEX2 \cdot df_z + PEX3 \cdot df_z^2) \cdot (1 - PEX4 \cdot \mathrm{sgn}(\kappa + S_{Hx})) \\
B \cdot C \cdot D &= F_z \cdot (PKX1 + PKX2 \cdot df_z) \cdot e^{-PKX3 \cdot df_z} \\
B &= \frac{B \cdot C \cdot D}{C \cdot D} \\
S_{Hx} &= (PHX1 + PHX2 \cdot df_z) \\
S_{Vx} &= F_z \cdot (PVX1 + PVX2 \cdot df_z)
\end{aligned} \tag{9}$$

the weighting function G for combined longitudinal force is:

$$\begin{aligned}
G(\alpha) &= \frac{\cos\left(C_\alpha \cdot \arctan(B_\alpha \cdot (\alpha + S_{H\alpha}))\right)}{\cos\left(C_\alpha \cdot \arctan(B_\alpha \cdot S_{H\alpha})\right)} \\
C_\alpha &= RCX1 \\
B_\alpha &= RBX1 \cdot \cos\left(\arctan(RBX2 \cdot \kappa)\right) \\
S_{H\alpha} &= RHX1
\end{aligned} \tag{10}$$

and for combined lateral force, where the coefficients for pure lateral characteristic are presented at Pacejka [11] as:

$$\begin{aligned}
D &= (PDY1 + PDY2 \cdot df_z) \\
C &= PCY1 \\
E &= (PEY1 + PEY2 \cdot df_z) \cdot (1 - PEY3 \cdot \mathrm{sgn}(\alpha + S_{Hy})) \\
B \cdot C \cdot D &= PKY1 \cdot F_{zo} \cdot \sin\left(2 \cdot \arctan\left(\frac{F_z}{PKY2 \cdot F_{zo}}\right)\right) \\
B &= \frac{B \cdot C \cdot D}{C \cdot D} \\
S_{Hx} &= (PHY1 + PHY2 \cdot df_z) \\
S_{Vx} &= F_z \cdot (PVY1 + PVY2 \cdot df_z)
\end{aligned} \tag{11}$$

the weighting function G for combined lateral force is:

$$\begin{aligned} G(\kappa) &= \frac{\cos\left(C_\kappa \cdot \arctan(B_\kappa \cdot (\alpha + S_{H\kappa}))\right)}{\cos\left(C_\kappa \cdot \arctan(B_\kappa \cdot S_{H\kappa})\right)} \\ C_\kappa &= RCY1 \\ B_\kappa &= RBY1 \cdot \cos\left(\arctan(RBY2 \cdot (\alpha - RBY3))\right) \\ S_{H\kappa} &= RHY1 \\ D_{v\kappa} &= D \cdot (RVY1 + RVY2 \cdot df_z) \cdot \cos\left(\arctan(RVY4 \cdot \alpha)\right) \\ S_{V\kappa} &= D_{v\kappa} \cdot \sin\left(RVY5 \cdot \arctan(RVY6 \cdot \kappa)\right) \end{aligned} \tag{12}$$

So the goal function is obtained computing the square difference sum (mean squared quadratic error) between the Magic Formula output and the test data and is expressed as follows:

$$\sum_{i=1}^{n} [Y^*_{pure}(X_i) - Y_{mes}(X_i)]^2 \tag{13}$$

at pure slip conditions, and:

$$\sum_{i=1}^{n} [Y^*_{combined}(X_i) - Y_{mes}(X_i)]^2 \tag{14}$$

at combined slip conditions.

When the objective is to fit the Magic Formula to test data at pure slip conditions, the optimization problem is given by:

$$\begin{aligned} &\min \ \sum_{i=1}^{n} [Y^*_{pure}(X_i) - Y_{mes}(X_i)]^2 \\ &where \quad X_i = \{\alpha_1, \alpha_2, \ldots, \alpha_i\} \ \vee \ X_i = \{\kappa_1, \kappa_2, \ldots, \kappa_i\} \\ &and \quad \{b_0, b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8, b_9, b_{10}, b_{11}, b_{12}, b_{13}\} \vee \\ &\{a_0, a_1, a_2, a_3, a_4, a_6, a_7, a_8, a_9, a_{11}, a_{12}, a_{17}\} \rightarrow Y^*_{pure} \\ &where\colon a_i \rightarrow F^X_{pure} \\ &\qquad\qquad b_i \rightarrow F^Y_{pure} \end{aligned} \tag{15}$$

where the parameters a_i are used when we use the pure longitudinal force F^X_{pure} to obtain Y^*_{pure} and the parameters b_i are used when we use the pure lateral force F^Y_{pure} to obtain Y^*_{pure}.

Under combined slip conditions the optimization problem is given by:

$$\begin{aligned}
&\min \ \sum_{i=1}^{n}[Y^*_{combined}(X_i) - Y_{mes}(X_i)]^2 \\
&where \quad X_i = \{\alpha_1, \alpha_2, \ldots, \alpha_i\} \ \vee \ X_i = \{\kappa_1, \kappa_2, \ldots, \kappa_i\} \\
&and \quad \left\{ \begin{array}{l} PDX1, PDX2, PCX1, PEX1, PEX2, PEX3, PEX4, PKX1, PKX2, \\ PKX3, PHX1, PHX2, PVX1, PVX2, RCX1, RBX1, RBX2, RHX1 \end{array} \right\} \vee \\
&\left\{ \begin{array}{l} PDY1, PDY2, PCY1, PEY1, PEY2, PEY3, PKY1, \\ PKY2, PHY1, PHY2, PVY1, PVY2, RCY1, RBY1, \\ RBY2, RBY3, RHY1, RVY1, RVY2, RVY4, RVY5, RVY6 \end{array} \right\} \rightarrow Y^*_{combined} \\
&where\!: \quad \{^{**}X^*\} \rightarrow F^X_{combined} \\
&\qquad\qquad \{^{**}Y^*\} \rightarrow F^Y_{combined}
\end{aligned} \tag{16}$$

where the parameters $\{^{**}\mathrm{X}^*\}$ are used when we use the combined longitudinal force $F^X_{combined}$ to obtain $Y^*_{combined}$ and the parameters $\{^{**}\mathrm{Y}^*\}$ when we use the combined lateral force $F^Y_{combined}$ to obtain $Y^*_{combined}$.

The objective is reduced to meet the parameters that define the Y^*_{pure} or $Y^*_{combined}$ function and that also minimize Equations (15) or (16) function at pure or combined slip conditions, respectively.

4. RESULTS

This section analyzes a set of results found applying the algorithm developed in the previous sections. All examples were programmed in a Pentium III 800 MHz and implemented in Matlab®.

The entire proposed algorithm, including the three operators, has been run sequentially following the scheme of Figure 3 where its simplicity is observed. However, it is easy to implement it in parallel, increasing the velocity of convergence significantly and constituting an additional advantage of the method. The proposed algorithm starts with the random generation of a starting population with *NP* individuals. The process to generate a new population of *NP* individuals is the following: the best $(\mathrm{X_{best}})$ and two random individuals $(\mathrm{X_{r1}}, \mathrm{X_{r2}})$ of the actual population are chosen and the disturbing vector (V) is formed as it is declared at Equation (3). Then the disturbing vector (V) and the *i* individual $(\mathrm{X_i})$ are crossed and mutate with a crossover probability (CP) and a mutation probability (MP) respectively, yielding a candidate $(\mathrm{X_i^N})$ of the following population. The following step consists of evaluating the candidate $(\mathrm{X_i^N})$ and the *i* individual $(\mathrm{X_i})$ at the goal function, the best of them will pass to be the *i* individual in the new population. This

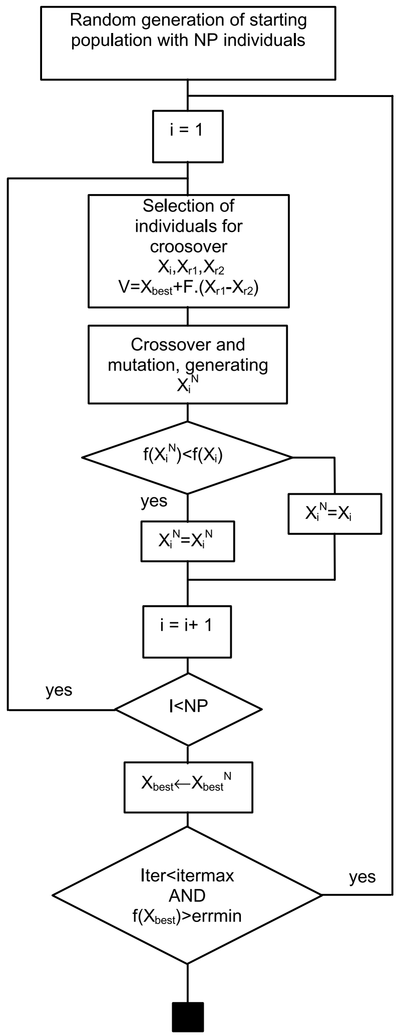

Fig. 3. Scheme of the entire algorithm for determination of Magic Formula parameters.

process is repeated to get the *NP* individuals of the new population, appearing a new best (X^N_{best}). New populations are generated until one of the two following conditions is satisfied: the number of iterations reaches the maximum number of iterations or the evaluation of the best of *N* population is lower than the minimum error.

To perform the test to the algorithm proposed, we need measurements of pure and combined longitudinal and lateral forces. The set of measured data used in this analysis is given for the following tyre type: Michelin XZA 11R22.5 $F_{z0} = 90$ kN and they were obtained in Palkovics and El-Gindy [12]. These set of measurements are shown in Table 1 (side forces at pure slip), Table 2 (longitudinal forces at pure slip). By multiplying the pure cornering and braking values with the coefficients given in Tables 3 and 4, the combined cornering and braking data values are obtained. The measurements shown in Tables are performed with zero camber angle.

To check the proposed method a set of previously stated measurements are used. And the sum squared error, given by Equations (15) or (16) is evaluated. The camber angle is not considered in these equations due to the fact that the used test data match with the test where the camber angle was zero. So, four cases are studied:

Table 1. Lateral force versus slip angle for different vertical loads [kN].

		α [deg]					
		0	1	2	4	8	12
F_z [kN]	8.754	0	1.575	2.803	4.727	6.741	7.966
	26.341	0	3.688	7.112	12.380	17.385	19.229
	41.677	0	4.168	7.919	15.004	22.923	25.423

Table 2. Longitudinal force versus slip for different vertical loads [kN].

		κ [–]						
		0	0.04	0.1	0.21	0.25	0.5	1
F_z [kN]	13.332	0	6.799	10.532	11.332	11.332	10.265	7.466
	26.663	0	8.799	18.398	21.064	21.064	19.198	12.799
	39.995	0	8.799	21.991	29.197	29.197	27.245	16.673

Table 3. Lateral roll-off table.

		κ [–]		
		0	0.25	1
α [deg]	0	1	0.38	0.06
	1	1	0.38	0.06
	2	1	0.38	0.06
	4	1	0.38	0.06
	8	1	0.50	0.09
	12	1	0.62	0.12

Table 4. Longitudinal roll-off table.

		α [deg]		
		0	4	12
κ [–]	0	1	1	0.44
	0.04	1	0.93	0.47
	0.1	1	0.87	0.52
	0.21	1	0.96	0.77
	0.25	1	0.97	0.78
	0.5	1	0.99	0.92
	1	1	1	0.98

Case 1, pure lateral force:

(a) The problem is defined by:

- The optimization process of the parameters uses the test data of Table 1. Only the measurements that correspond to those obtained when the vertical load $F_Z = 8.754$ kN and $F_z = 41.677$ kN were used. During the training process, the symmetrical data set was produced by inverting the original measured data, provided in Table 1.
- The parameters to be optimized are: $[a_0, a_1, a_2, a_3, a_4, a_6, a_7, a_8, a_9, a_{11}, a_{12}, a_{17}]$ defined at Equation (15).
- Parameters of the algorithm: $NP = 50$, $CP = 0.6$, $MP = 0.1$, $F = 0.4$, $itermax = 1000$.
- The values of initial population are randomly comprised between [0, 1].

(b) The best parameters to fit the curves defined in Table 1 are:

$a_0 = 0.384\,[\,]$	$a_1 = -0.014\,[1/kN]$	$a_2 = -2.228\,[\,]$
$a_3 = 241.4\,[kN/rad]$	$a_4 = 47.72\,[kN]$	$a_6 = 0.642\,[1/kN]$
$a_7 = 4.565\,[\,]$	$a_8 = 12.137\,[rad/kN]$	$a_9 = -3.547\,[rad]$
$a_{11} = 4.35e-6\,[\,]$	$a_{12} = 2.01e-4\,[kN]$	$a_{17} = 0.945\,[\,]$

The evolution of the goal function along the iterations is shown in Figure 4. While Figure 5 shows the last optimization result. Notice how the best parameters obtained fit the F_y values properly when $F_z = 26.341$ kN. The algorithm was not trained when the vertical load was this.

Case 2, pure longitudinal force:

(a) The problem is defined by:

- The optimization process of the parameters uses the test data of Table 2. Only the measurements that correspond to those obtained when the vertical load

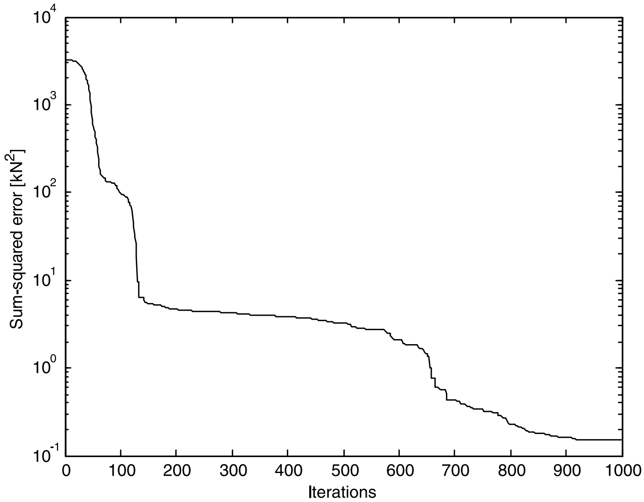

Fig. 4. Sum-squared error during the training of pure lateral force.

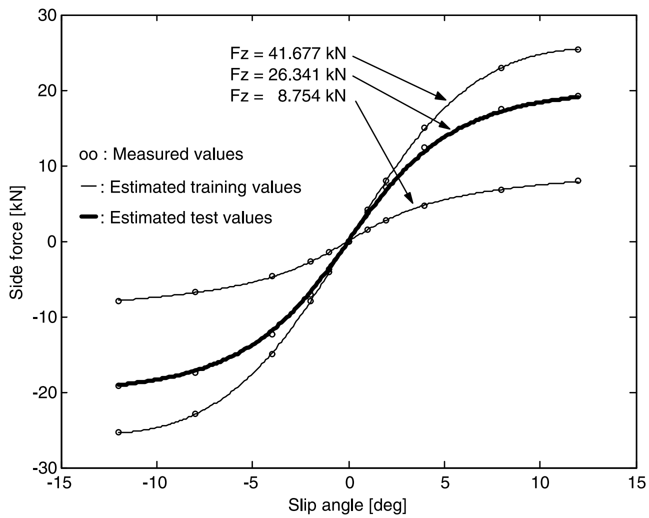

Fig. 5. Estimated and measured side forces for three different vertical loads.

$F_z = 13.332\,\text{kN}$ and $F_z = 39.995\,\text{kN}$ were used. During the training process, the symmetrical data set was produced by inverting the original measured data, provided in Table 2.

– The parameters to be optimize are: $[b_0, b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8, b_9, b_{10}, b_{11}, b_{12}, b_{13}]$ defined at Equation (15).
– Parameters of the algorithm: NP = 50, CP = 0.6, MP = 0.1, F = 0.4, itermax = 1000.
– The values of initial population are randomly comprised between [0, 1].

(b) The best parameters to fit the curves defined in Table 2 are:

$b_0 = 2.139\,[\,]$	$b_1 = 0.0045\,[1/\text{kN}]$	$b_2 = -0.934\,[\,]$	$b_3 = 1.971\,[1/\text{kN}]$
$b_4 = 6.081\,[\,]$	$b_5 = 0.0654\,[1/\text{kN}]$	$b_6 = -0.0014\,[1/\text{kN}^2]$	$b_7 = 0.040\,[1/\text{kN}]$
$b_8 = 2.229\,[\,]$	$b_9 = 9.716\,[1/\text{kN}]$	$b_{10} = 5.626\,[\,]$	$b_{11} = 3.2\text{e}-6\,[\,]$
$b_{12} = -8.7\text{e}-5\,[\text{kN}]$	$b_{13} = 0.649\,[\,]$		

Figure 6 shows the last optimization result. Notice how the best parameters obtained fit the F_x values properly when $F_z = 26.663$ kN. The algorithm was not trained when the vertical load was the latter ($F_z = 26.663$ kN).

Case 3, combined lateral force:

(a) The problem is defined by:

– The set of measurements that the optimization process uses is expressed multiplying the pure lateral force data in Table 1 when $F_z = 8.754$ kN by the coefficients in Table 3, when the longitudinal slips are: $\kappa = [0\ 0.25\ 1]$. During

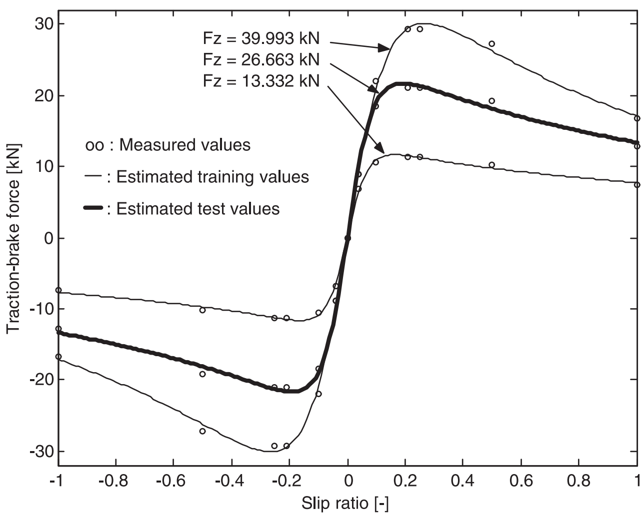

Fig. 6. Estimated and measured longitudinal forces for three different vertical loads.

the training process, the symmetrical data set was produced by inverting the originally measured data, provided in the Tables 1 and 3.

- Parameters to optimize are:
  [PDY1, PDY2, PCY1, PEY1, PEY2, PEY3, PKY1, PKY2, PHY1, PHY2, PVY1, PVY2, RCY1, RBY1, RBY2, RBY3, RHY1, RVY1, RVY2, RVY4, RVY5, RVY6] of Equation (16).
- Parameters of the algorithm: NP = 50, CP = 0.6, MP = 0.1, F = 0.4, itermax = 1000.
- The values of initial population are randomly comprised between [0, 1].

(b) The best parameters to fit the curves are:

PDY1 = −1.448	PDY2 = 0.9025	PCY1 = 0.4091	PEY1 = 2.5795
PEY2 = 2.182	PEY3 = −0.00463	PKY1 = 0.109	PKY2 = 1.2704
PHY1 = 5.7623	PHY2 = 4.3024	PVY1 = 0.6211	PVY2 = 0.6877
RCY1 = 1.039	RBY1 = 10.918	RBY2 = 0.1395	RBY3 = 0.184
RHY1 = −0.0272	RVY1 = 0.6304	RVY2 = 0.6924	RVY4 = 4.051
RVY5 = −0.0174	RVY6 = −0.00159		

Figure 7 shows the last optimization result. This figure shows the side force for three different longitudinal slips.

Case 4, combined longitudinal force:

(a) The problem is defined by:

- The set of measurements that the optimization process uses is expressed multiplying the pure longitudinal force data in Table 2 when $F_z = 13.332\,kN$ by the coefficients in Table 4, when the slip angles are: $\alpha = [0^\circ\ 4^\circ\ 12^\circ]$. During the training process, the symmetrical data set was produced by inverting the originally measured data, provided in the Tables 2 and 4.
- Parameters to optimize are:
  [PDX1, PDX2, PCX1, PEX1, PEX2, PEX3, PEX4, PKX1, PKX2, PKX3, PHX1, PHX2, PVX1, PVX2, RCX1, RBX1, RBX2, RHX1] of Equation (16).
- Parameters of the algorithm: NP = 50, CP = 0.6, MP = 0.1, F = 0.4, itermax = 1000.
- The values of initial population are randomly comprised between [0, 1].

(b) The best parameters to fit the curves are:

PDX1 = 1.749	PDX2 = 1.036	PCX1 = 1.847	PEX1 = 0.5476
PEX2 = 0.3665	PEX3 = 0.48532	PEX4 = 0.22936	PKX1 = −1.014
PKX2 = −3.206	PKX3 = 2.5009	PHX1 = −3.0346	PHX2 = −0.2587
PVX1 = 0.8685	PVX2 = 1.0191	RCX1 = −1.9344	RHX1 = 2.0564
RBX1 = −0.0494	RBX2 = 6.6232		

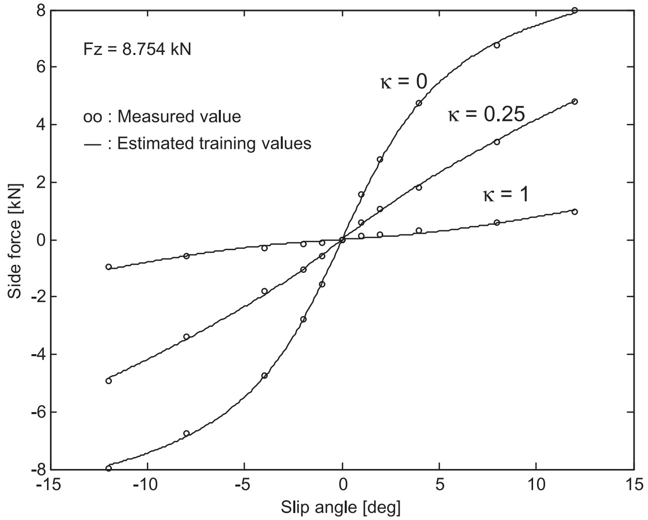

Fig. 7. Estimated and measured side forces for three different longitudinal slips.

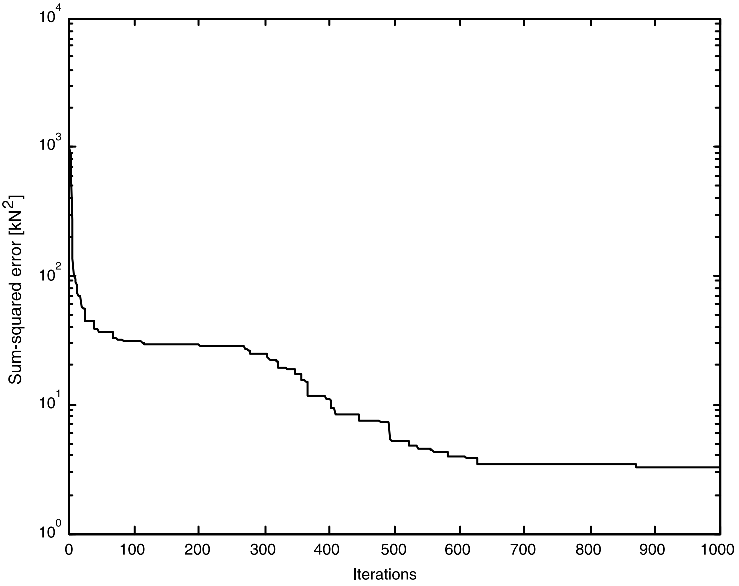

Fig. 8. Sum-squared error during the training of combined side force.

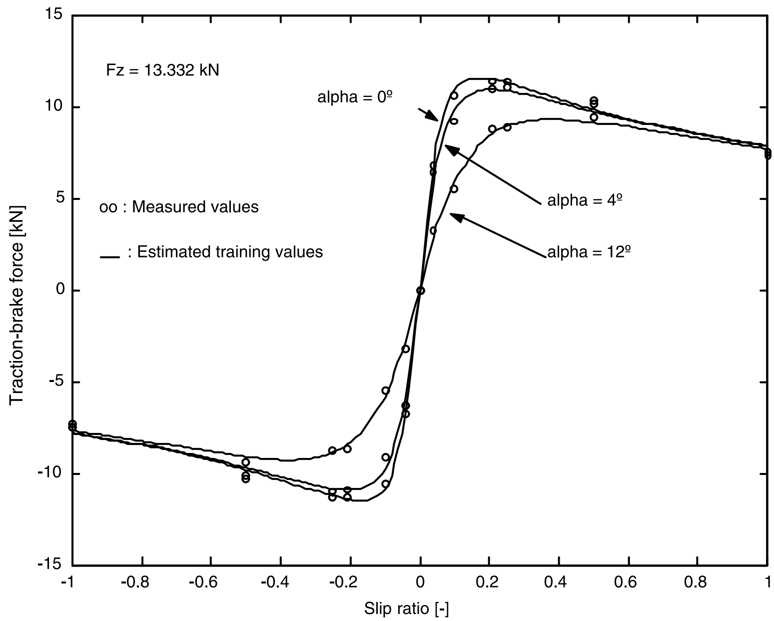

Fig. 9. Estimated and measured longitudinal forces for three different side slips.

The evolution of the goal function along the iterations is shown in Figure 8. Moreover Figure 9 shows the last optimization result with the three longitudinal force curves for three slip angle cases.

5. DISCUSSION ABOUT THE METHOD

The first two solved cases deal with pure lateral and longitudinal forces with the Magic Formula published in [3]. The first case of the two shows the goal function evolution with iterations (see Fig. 4), meeting an optimum with little error. It was verified that, using the Simplex search algorithm of Nelder and Mead [13], if initial values of the parameters were far from the optimum, the final error was bigger than the error made before and if the initial values were near the optimum, the final error was of the same magnitude as the error made by the proposed method, but continued being bigger. So the disadvantage of this analytical optimization method is that due to the big quantity of parameters to be optimized, to look for a suitable initial starting condition can become a heavy task. However, the proposal method here does not need to know the physics of the parameters to be optimized, so the initial values of parameters are indifferent, for instance, in this paper the values are comprised between [0, 1].

To build the goal function in the second of the two cases the Magic Formula proposed in [4, 11] has been used. Here the number of parameters is bigger than in the first two cases. For case 4, the error in the optimization process is of the same magnitude as the error made in pure lateral force (see Fig. 8), but slightly bigger than the error made in pure lateral force. However if the error is compared with the error made in the Neuro-Tyre optimization process [12], the difference is ten times less. Also, if we compare the error with the one made in the Nelder and Mead optimization method when the initial conditions are chosen in a random way, the error is bigger than the error in the proposed method. And in the Neuro-Tyre method the error is of the same magnitude as in the Nelder and Mead method.

In each case the best solution was found for a population size of 100, probabilities for crossover and mutation at 0.6 and 0.1 respectively, and the F factor of the disturbing vector of the selection equals 0.4. The initial values used by the algorithm are randomly chosen by the computer code in range $[0\ 1]$. But it was verified that the reached solution was always very similar for those cases and any other tested. This fact points out that the method escapes local minimums when the number of evaluations is large enough.

It is observed that the number of iterations required by genetic algorithms is generally much larger than the iterations used by gradient based methods. But the evaluation of the function is much more complex for analytical methods, which need the computation of first order derivatives of the objective functions. Thus the computation time of the genetic algorithms is lower in general.

6. CONCLUSIONS

The Magic Formula Tyre Model for the description of steady tyre behavior in vehicle dynamics simulations is very accurate and widely used, but the determination of the Magic Formula parameters may be difficult. This paper deals with a new method based on evolutionary techniques to determine the parameters of the Magic Formula.

In this paper, the used method, which is suitable for the determination of the Magic Formula parameters, was verified. This method was implemented to get the parameters of two different versions of the Magic Formula. The braking and traction force and lateral force, at pure and combined maneuvers were obtained successfully. The Magic Formula was fitted to test data for all the cases. The simplicity of the algorithm implementation and the option of starting the optimization process with any initial values for the parameters, were the main advantages of the proposed method. So the required experience for the method users diminishes significantly and substantially.

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