Spiral wave break-up and planar front formation in two-dimensional reactive–diffusive media with straining

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Abstract

The propagation of spiral waves in two-dimensional reactive–diffusive media subject to a velocity field with straining is studied numerically. It is shown that, depending on the magnitude of the strain rate, the effects of straining on wave propagation can be classified in three regimes: the no break-up, the transitional and the break-up regimes. In the no break-up regime, the spiral wave preserves its integrity, whereas, in the transitional one, the spiral wave is deformed and transformed into a curved front which propagates almost along one of the principal directions of the strain rate tensor. In the break-up regime, spiral waves are broken up into many spiral waves, and planar fronts aligned with the principal direction of the strain rate tensor corresponding to elongation/stretching are formed. These fronts may be split into two new ones or are bands, the thickness of which decreases as the strain rate is increased.

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1. Introduction

Spiral waves in two-dimensional reactive–diffusive and excitable media arise in a variety of fields such as chemistry, physics, biology, combustion, etc., and their stability depends on the convective and diffusive transport, because
transport provides the coupling between spatially separated elements and brings about the front propagation [1]. Of the two kinds of transport, the diffusive one has been more extensively studied than the convective transport, especially in biological media [2]. Convective transport was studied by Wellner et al. [3] who considered the drift of stable, meandering spiral waves in a singly diffusive FitzHugh–Nagumo (FN) medium caused by a weak time-independent gradient or convection in the fast-variable equation, showed by means of perturbation methods the equivalence between gradient and convective perturbations, and proposed a semi-empirical solution to the drift of spiral waves that depends on the period of rotation and the value of the fast variable at the center of the spiral wave.

Biktashev et al. [4,5] considered an excitable medium in two dimensions with a cubic non-linearity given by the FitzHugh–Nagumo system and a shear characterized by a velocity field in the x-direction which is either a linear or a sinusoidal function of the y-coordinate, and showed that the shear can distort and then break spiral waves. Such breaks were found to result in a chain reaction of spiral wave births and deaths. The velocity fields employed by Biktashev et al. [4] are one-directional and solenoidal, but not irrotational, and they do not satisfy the no-penetration condition at the boundaries of the domain; in fact, these authors used periodicity conditions for the sinusoidal velocity field. Biktashev et al. [6] considered excitable media with straining and showed that solitary waves require the velocity gradient to exceed a certain threshold to break up, while the breaking of repetitive wave trains happens for arbitrarily small velocity gradients.

The break-up of spiral waves as well as the formation of thick fronts and the formation of tiles in reactive–diffusive media in two-dimensional rectangular domains with advection have been explained in terms of the straining of the wave front and convection-induced anisotropy in non-solenoidal velocity fields [7–10] in an analogous manner to that employed in combustion theory to explain flame extinction [1]. Moreover, the boundary conditions on the velocity field also play a role in determining the spiral wave dynamics and its break-up. When the velocity field satisfies the no-slip condition, the boundaries of the domain are stagnation lines where convective effects are nil, whereas the velocity field is parallel to the boundaries when the no-penetration condition is employed. In addition, the velocity fields employed in previous two-dimensional numerical studies were sinusoidal and were characterized by space-dependent strain rate tensors [7,9,10]; therefore, complex patterns were observed at high spatial frequencies due to the straining of the reaction fronts.

In order to assess the influence of flow straining on wave propagation in two-dimensional reactive–diffusive media, it is convenient to employ simple velocity fields which satisfy the incompressible flow condition, so that these fields have constant strain rate tensors and do not affect the reaction rate through volumetric expansion/compression. The simplest flow fields which
satisfy these restrictions are linear in the Cartesian coordinates $x$ and $y$, but they do not satisfy the no-slip and no-penetration conditions at the boundaries of the domain.

In this paper, a numerical study of reactive–diffusive media governed by the two-equation Oregonator model of the Belousov-Zhabotinsky (BZ) system in two-dimensional, stationary, convective/advective, solenoidal flow fields with straining is presented. The velocity components are linear functions of the Cartesian coordinates $x$ and $y$, and are assumed to be the same for both the fast and slow variables, i.e., the same for both the activator and the inhibitor. The main objective of the study is to determine the effects of flow straining on the propagation, distortion and break-up of spiral waves in two-dimensional reactive–diffusive media. Since the velocity fields considered in this paper are solenoidal, they do not introduce compressible effects which would alter the shape and speed of propagation of the spiral wave [7,9,10].

2. Governing equations

The numerical study presented here is based on the BZ reaction which is often modelled by the two-equation Oregonator model [11] and may be written as

$$\frac{\partial u}{\partial t} + \mathbf{v} \cdot \nabla u = d_u \nabla^2 u + F_u, \quad (1)$$

$$\frac{\partial v}{\partial t} + \mathbf{v} \cdot \nabla v = d_v \nabla^2 v + F_v, \quad (2)$$

where $t$ is time, $u$ and $v$ denote the concentrations of the activator and the inhibitor, respectively, $d_u$ and $d_v$ are the (constant) diffusion coefficients for $u$ and $v$, respectively, $\mathbf{v} = (v_x, v_y)^T$ is the velocity field where the subscripts $x$ and $y$ denote the corresponding axes, and the source terms in Eqs. (1) and (2) can be written as

$$F_u = \frac{1}{\epsilon} \left( u - u^2 - f v \frac{u - q}{u + q} \right), \quad F_v = u - v, \quad (3)$$

where $\epsilon = 0.01$, $f = 1.4$ and $q = 0.002$ and are the same as those employed in the BZ model [7–10]. The (homogeneous) two-equation BZ model employed in this study has only two critical points, i.e., the nullclines of $u$ and $v$ intersect at only two points, and, therefore, does not correspond to an excitable model where the nullclines of the activator and inhibitor intersect at a unique steady state and a finite perturbation may bring the system across the unstable branch of the activator nullcline to the high concentration stable branch of the activator where this accumulates rapidly and the system returns to the steady state.
by jumping from the high concentration stable branch to the lower concentration stable branch of the activator.

Since the interest in this paper lies in the effects of flow straining on wave propagation in two-dimensional reactive–diffusive media, it is assumed that \( d_u = 1 \) and \( d_v = 0.6 \), and it is known that, for these values, the two-equation Oregonator model has spiral wave solutions when \( v = 0 \) and homogeneous Neumann boundary conditions are imposed on all the boundaries of the domain.

Eqs. (1) and (2) were solved in the (square) spatial domain \( \Omega = [-L_x/2, L_x/2] \times [-L_y/2, L_y/2] \) where \( L_x = L_y = 15 \), subject to homogeneous Neumann boundary conditions on all the boundaries and to the following initial condition:

\[
\begin{align*}
u &= 0 \quad \text{for } 0 < \theta < 0.5; \quad \nu = q(f + 1)/(f - 1) \quad \text{elsewhere}, \\
v &= q \frac{f + 1}{f - 1} + \frac{\theta}{8\pi f},
\end{align*}
\]

where \( \theta \) is the angle with respect to the origin of coordinates measured counterclockwise from the positive \( x \)-axis. In the absence of convection, this initial condition results in the formation of a spiral wave which rotates counterclockwise.

The use of homogeneous Neumann boundary conditions in the present study is, of course, an idealization which implies that the diffusive fluxes at the boundaries are nil, whereas there is convection at these boundaries owing to the non-zero velocity component normal to the boundary; therefore, there may be a non-zero flux of activator and inhibitor normal to the boundaries of the domain. More physically plausible boundary conditions should account for the continuity of the total (convective + diffusive) fluxes normal to the boundaries which, in turn, depend on the fluxes outside the domain. Alternatively, one could employ boundary conditions analogous to Newton’s cooling law in heat transfer, or Danckwerts’ boundary conditions which neglect the diffusive transport but require the specification of the convective fluxes outside the domain. In a previous study [12], homogeneous Danckwerts’ boundary conditions were employed and the results showed that spiral waves may be broken up by the straining field and may be transformed into curved fronts that propagate along one of the principal directions of the strain rate tensor; therefore, a comparison between the results presented here and those of the author [12] will indicate the effects that the boundary conditions have on the break-up of spiral waves in two-dimensional reactive–diffusive media subject to straining.

The velocity field employed in this study can be written as

\[
v_x = -\gamma y, \quad v_y = -\gamma x,
\]
where $-\gamma = \frac{1}{2} \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right)$ is the strain rate. The strain rate tensor, i.e.,

\[ e = \frac{1}{2} (\nabla v + (\nabla v)^T), \]

where the superscript $T$ denotes transpose, has eigenvalues equal to $+\gamma$ and $-\gamma$ and associated eigenvectors $(1, -1)^T$ and $(1, 1)^T$, respectively, which correspond to the principal directions of the strain rate tensor. The direction $(1, 1)^T$ coincides with that of the velocity field employed in this study.

The velocity field given by Eq. (6) is irrotational, i.e., $\nabla \times v = 0$, and solenoidal, i.e., $\nabla \cdot v = 0$, and, therefore, the convective terms in Eqs. (1) and (2) can be written in strong conservation-law form, i.e., $v \cdot \nabla u = \nabla \cdot (uv)$, and do not introduce compressible effects that would affect the effective reaction rate. Furthermore, for the Cartesian domain considered in this study, the velocity field employed here does not satisfy the no-slip and no-penetration conditions at the boundaries, and the flow has a stagnation point at $(x, y) = (0, 0)$; it is, therefore, an idealization. Flow fields that satisfy the incompressibility and the irrotational flow conditions, and the no-penetration and no-slip boundary conditions do exist but are more complicated than the linear one employed in this study [7]. In addition, the use of homogeneous Neumann boundary conditions implies that there is a (net) convective flux at the boundaries if the concentration at the boundaries is different from zero. In the numerical study of Biktashev et al. [6], there is no flux at the horizontal walls, while there is a flux at the inlet and outlet on account of the periodic boundary conditions employed by these authors.

Eqs. (1) and (2) were solved numerical by means of an implicit, time-linearized, second-order accurate (in both space and time) finite difference method [13]. This method factorizes the elliptic equations that result upon discretization of time at each time level, into two one-dimensional boundary value problems and employs an iterative technique to account for the approximate factorization errors. $\gamma$ was set to zero from $0 \leq t \leq 100$, and, at $t = 100+$, the straining flow was switched on. Computations were performed on a $102 \times 102$ point equally spaced mesh and a time step of $10^{-4}$. Computations were also performed with equally spaced meshes of $202 \times 202$ and $502 \times 502$ points and different time steps, in order to ensure that the results were independent of both the number of grid points and the time step. In the next section, some sample results obtained with $102 \times 102$ point equally spaced meshes and a time step of $10^{-4}$ are presented.

3. Results

In this section, some sample results on the propagation of spiral waves in two-dimensional reactive–diffusive media subject to velocity fields with straining are presented; the straining was varied from $\gamma = -0.5$ to 0.5 in steps...
of 0.05. For $\gamma = 0$, i.e., in the absence of straining, a spiral wave was observed, and the tip of this wave rotated about the center of the domain. For $\gamma \neq 0$, the effects of straining on the propagation of spiral waves can be classified into three different regimes: the no break-up regime for $|\gamma| \leq 0.05$, the transitional regime for $0.05 < |\gamma| < 0.15$, and the break-up regime for $|\gamma| \geq 0.15$. In the no break-up regime, the spiral wave was deformed and stretched in the direction $(1, -1)^T$ for $\gamma > 0$ and in the direction $(1, 1)^T$ for $\gamma < 0$, but did not break up. For $|\gamma| \leq 0.05$, the spiral wave preserved its integrity, whereas, for $\gamma = 0.10$, i.e., in the transitional regime, it was observed that the spiral wave was stretched in the direction $(1, -1)^T$ by the flow field until about $t = 120$. For $120 < t \leq 140$, the spiral wave underwent deformations such as those shown in Fig. 1 until the curved arm that connects the top and left boundaries of the domain in the last frame of this figure approached the right and bottom boundary. Later on, no spiral wave was observed; instead, a periodic behavior such as the one illustrated in Fig. 2 was observed. This behavior indicates that a high-concentration curved front is formed at the upper left corner and propagates towards the right and bottom boundaries of the domain; this curved front reaches the

Fig. 1. Concentration of the activator $u$ at (from left to right, from top to bottom) $t = 124, 126, 128, 130, 132, 134, 136, 138$ and $140$ for $\gamma = 0.10$. 

bottom boundary before the right one, and is much thicker than the spiral wave observed without straining.

For larger values of $|\gamma|$, i.e., $|\gamma| > 0.15$, the spiral wave broke up, although the time for a full break-up and the patterns that were observed after imposing the velocity field and after break-up were found to depend on $|\gamma|$; for example, the results presented in Fig. 3 which correspond to $\gamma = -0.20$ indicate that the spiral wave is initially stretched in the principal direction of the strain rate tensor corresponding to elongation/stretching (second and third frames of Fig. 3), then thins and breaks up (fourth and fifth frames), and later there is merging and a distorted spiral wave arm is formed (ninth frame). At much larger times, the results presented in Fig. 4 indicate that a high concentration front emerges from the lower left corner and propagates towards the upper right corner of the domain forming an almost planar front that propagates in the direction $(1, -1)^T$ (frames 3 and 4). The activator concentration in this front is high, but this high concentration region is split into two new regions (frames 6–8) which propagate towards the upper left and lower right corners of the domain and yield high concentration regions of an almost triangular shape.

Fig. 2. Concentration of the activator $u$ at (from left to right, from top to bottom) $t = 164.2$, 164.3, 164.4, 164.5, 164.6, 164.7, 164.8, 164.9 and 165 for $\gamma = 0.10$. 
near these corners. The behavior just described repeats itself in a periodic fashion for $t \leq 140$.

Although not shown here, for $|\gamma| = 0.15$, a similar behavior to the one shown in Fig. 4 was found, except that the fronts formed after wave splitting had a larger curvature than the ones of that figure, and the spiral wave took a longer time to break up.

For $\gamma = 0.25$, the spiral wave is initially stretched by the velocity field and then breaks up into a number of smaller spiral waves as indicated in the third, fifth and seventh to ninth frames of Fig. 5. At later times, the results shown in Fig. 6 indicate that the activator concentration exhibits planar fronts aligned with the principal direction of the straining rate tensor corresponding to stretching/elongation, which propagate from the upper right to the lower left corner. Although not shown in Fig. 6, these fronts are in fact bands with leading and trailing edges as illustrated in the fifth and sixth frames of Fig. 7 which corresponds to $\gamma = 0.45$ and indicate that the activator concentration is higher at the front than at the back of the planar front in frames 3–6.
Fig. 7 also indicates that the activator’s concentration is initially high and almost uniform in the upper right corner, and decreases sharply to a low value on the lower left corner. The almost plane front propagates from the upper right to the lower left corner rather quickly and the activator’s concentration exhibits a layered structure behind the leading edge of the front. The front becomes a band where the activator’s concentration is high when the front has advanced beyond the diagonal that connects the upper left and the lower right corners, and then acquires a triangular shape which becomes curved near the left and bottom boundaries. The behavior just described repeats itself in a periodic fashion.

Similar trends to those shown in Fig. 7 have been observed for $\gamma = 0.50$ and 1. For $\gamma = 1$, the band observed in the fifth and sixth frames of Fig. 7 was thinner and exhibited more curvature at the left and bottom boundaries than that for $\gamma = 0.5$.

For both positive and negative values of $\gamma$, it was found that the time required to break the spiral wave decreases as the straining rate is increased. For example, the end stages of the break-up of spiral waves were observed at
approximately $t = 112, 120, 114$ and $108$ for $\gamma = -0.15, -0.25, -0.35$ and $-0.45$, respectively.

The results presented in Figs. 4 and 5 differ from the two-dimensional numerical results performed by Biktashev et al. [6] on excitable media subject to a unidirectional velocity field, i.e., $u = -xy$ and $v = 0$, no-flux conditions at the horizontal walls and periodic boundary conditions in the horizontal direction. These authors showed that solitary waves require the velocity gradient to exceed a certain threshold to break up, while the breaking of periodic wave trains happens for arbitrarily small velocity gradients. Our numerical studies, however, indicate that there is threshold strain rate above which spiral waves break up, and this behavior is in accord with that observed by Biktashev et al. [6] for solitary waves, but not for spiral waves. Moreover, our numerical studies have shown that (1) there is a transitional regime where spiral waves do not break up, but are highly deformed by the straining flow field and are transformed into thick curved reaction fronts, (2) the initial deformation of the spiral wave and its straining occur along the principal direction of the strain rate tensor corresponding to elongation, i.e., $(1, -1)^T$ for $\gamma > 0$, (3) almost planar fronts and
planar bands are observed for straining rates higher than a threshold, these fronts propagate along the direction $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$, i.e., along the direction of the velocity field, and were not reported by Biktashev et al. [6], (4) the orientation of the waves predicted by Biktashev et al. [6] is less than $90^\circ$ which is the value predicted in our studies, (5) front splitting is observed at moderate straining rates in the break-up regime, and (6) the thickness of the planar bands observed in the break-up regime decreases as the straining rate is increased. The differences between the results presented here and those of Biktashev et al. [6] may be due to (1) the different “chemistry” (Biktashev et al. [6] used the FitzHugh–Nagumo system whereas here we employed the BZ system), (2) the different velocity fields (Biktashev et al. [6] used a unidirectional, incompressible but rotational flow, whereas we have employed a two-dimensional, solenoidal and irrotational velocity field), and (3) the different boundary conditions (Biktashev et al. [6] used no-flux conditions in the $y$-direction and periodic boundary conditions in the $x$-direction, whereas we employed zero diffusive fluxes at the boundaries). Furthermore, a comparison between the results presented here and those of the author [12] who considered no-flux boundary conditions

Fig. 6. Concentration of the activator $u$ at (from left to right, from top to bottom) $t = 164.16, 164.24, 164.32, 164.40, 164.48, 164.56, 164.64, 164.72$ and $164.80$ for $\gamma = 0.20$. 
indicates, that the boundary conditions play a paramount role in determining the pattern formation in two-dimensional excitable media subject to straining once the spiral waves are broken up by the advective flow field.

The values of $u$ and $v$ were monitored at $(x, y) = \left(\frac{\pi}{L_x}, \frac{\pi}{L_y}\right)$, $(\frac{\pi}{L_x} + 51\delta, \frac{\pi}{L_y})$, $(\frac{\pi}{L_x} + 51\delta, \frac{\pi}{L_y} + 56\delta)$, $(\frac{\pi}{L_x} + 61\delta, \frac{\pi}{L_y} + 61\delta)$, $(\frac{\pi}{L_x} + 71\delta, \frac{\pi}{L_y} + 71\delta)$, $(\frac{\pi}{L_x} + 81\delta, \frac{\pi}{L_y} + 81\delta)$, $(\frac{\pi}{L_x} + 91\delta, \frac{\pi}{L_y} + 91\delta)$ and $(\frac{\pi}{L_x} + 96\delta, \frac{\pi}{L_y} + 96\delta)$ where $\delta = 15/101$ as functions of time every one hundred time steps. The values of $u$ at these monitor locations are characterized by narrow spikes where the activator concentration raises sharply to its maximum value and then decreases sharply to almost a nil value once a periodic behavior is achieved, i.e., for $t > 140$, whereas the inhibitor’s concentration raises sharply and then decreases much more slowly to a background value which is different from zero. The phase diagrams of $u$ vs. $v$ at the monitor locations for $t > 140$ consists of a single closed curve, and $v$ is in the range $(0.01, 0.21)$. The Fourier spectra of both $u$ and $v$ at the monitor locations for $t > 140$ contain a single peak which corresponds to the period of the oscillations. This period ($\pm 0.01$) or distance between successive peaks of $u$ at

![Fig. 7. Concentration of the activator $u$ at (from left to right, from top to bottom) $t = 164.12$, 164.24, 164.36, 164.48, 164.60, 164.72, 164.84, 164.96 and 165.08 for $\gamma = 0.45$.](image-url)
\((-L_x/2 + 56\delta, -L_y/2 + 56\delta)\) is equal to 1.60, 1.62, 4.25, 4.24, and 4.21 for \(|\gamma| = 0.01, 0.05, 0.1, 0.15\) and 0.20 \(\leq \gamma \leq 1\), whereas the maximum values of \(u\) at the monitoring locations were found to be approximately equal to 0.7312, 0.7613, 0.9198, 0.9227 and 0.9184 for \(|\gamma| = 0.01, 0.05, 0.1, 0.15\) and 0.20 \(\leq |\gamma| \leq 1\); therefore, both the period and the maximum activator concentration are increasing functions of \(|\gamma|\) in the no break-up regime, first increase and then decrease in the transitional regime, and remain constant and, therefore, are independent of \(|\gamma|\) in the break-up regime.

Although not shown here, the results obtained for \(\gamma < 0\) are analogous to those reported before, except that the planar fronts formed in the break-up regime are parallel to the diagonal joining the upper right and the lower left corners of the domain. As shown in the previous section, for \(\gamma < 0\), there is stretching along the principal direction \((1, 1)^T\) and compression along \((1, -1)^T\); therefore, the results obtained indicate that the planar fronts align themselves with the principal direction along which there is stretching and propagate along the direction along which there is compression.

As indicated previously [7,9,10], the motion in the vicinity of a point in a two-dimensional reactive–diffusive medium can be resolved into a uniform translation \(v\), a rigid body rotation with angular velocity equal to \(\frac{1}{2} \nabla \times v\) and a pure straining motion. The first two do not have any effect on the internal structure of a locally planar wave front, whereas the third one is described by the symmetric strain rate tensor \(e\). For locally planar wave fronts, the most important decomposition of \(e\) at stagnation points is associated with \(b = n \cdot e \cdot n\) where \(n\) is the unit vector normal to the front. In a two-dimensional stagnation point flow, the rate of change of the transverse component of velocity with transverse distance is \(t \cdot e \cdot t\) where \(t\) is the unit vector tangent to the front and this is equal to \(-b\). This means that a decrease in the normal mass flux with distance through the front, i.e., \(b < 0\), is reflected in a net transverse outflow, i.e., \(t \cdot e \cdot t > 0\). In combustion theory, this outflow is referred to flame stretch [1], and the stretching can be written as \(-n \cdot \nabla \times (v \times n) = -(n \cdot \nabla)(v \cdot n) + \nabla \cdot v\). For the solenoidal velocity field employed in this paper, the last term of this expression is nil, and the front aligns itself with the principal direction of the strain rate tensor corresponding to stretching or elongation, while it propagates along the other principal direction which is associated with compression, i.e., for \(\gamma > 0\), stretching occurs along \((1, -1)^T\), and compression along \((1, 1)^T\).

A comparison between the results presented here and those previously reported by the author [7,9,10] who used sinusoidal velocity fields with complex space-dependent strain rate tensors indicates that flow straining plays a paramount role in determining wave propagation in two-dimensional reactive–diffusive media; flow straining distorts the reaction front and aligns it with the principal direction of the deformation rate tensor corresponding to stretching. Moreover, previous numerical studies of the effects of advection on wave...
propagation in two-dimensional reactive–diffusive media [7,9,10] satisfy either the no-penetration condition, i.e., \( \mathbf{n} \cdot \mathbf{v} = 0 \), or the no-slip condition, i.e., \( \mathbf{n} \cdot \mathbf{v} = \mathbf{t} \cdot \mathbf{v} = 0 \), where \( \mathbf{n} \) and \( \mathbf{t} \) denote the unit vector normal to and tangential, respectively, to the boundary and \( \mathbf{v} \) denotes the velocity vector, and, therefore, the advective flux normal to the boundary is zero when the no-penetration condition is imposed at the boundaries. By way of contrast, the advective flux at the boundaries is not nil in the study presented here.

4. Conclusions

The effects of flow straining on wave propagation in two-dimensional reactive–diffusive media have been studied numerically with a simple flow field whose velocity components are linear functions of the Cartesian coordinates, and it has been shown that there are three regimes. In the break-up regime, the flow straining destroys spiral waves and results in almost planar fronts which are aligned with the principal direction of the strain rate tensor corresponding to stretching; these fronts may split into two different fronts or form bands, the thickness of which decreases as the straining rate is increased. Below this threshold, spiral waves may preserve their integrity in the no break-up regime, or may be deformed and transformed into curved thick fronts in the transitional regime. The period of the wave patterns has been found to increase in the no break-up regime, increase and then decrease in the transitional regime, and remain constant in the break-up regime as the straining rate was increased. The results presented in this paper indicate that the straining rate must exceed a threshold value for spiral waves to break up, whereas previous two-dimensional studies in excitable media indicate that there is a threshold for the breaking of solitary waves and the breaking of repetitive wave trains occurs for arbitrarily small values of the velocity gradient. Our results regarding the orientation of the resulting wave after break-up also differ from previous ones, presumably because of the differences in geometry, reaction terms, velocity field, and boundary conditions.

It has also been found that the destruction of spiral waves in two-dimensional reactive–diffusive media subject to straining can be explained locally in terms of the strain rate tensor and the fluxes of the activator and inhibitor at the reaction front.

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References